WHAT DO UNIONS DO (TO NONUNION WORKERS)?

Pablo Ruiz Verdú*1

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Keywords: Union Threat, Employment Determination, Wage Bargaining Models

JEL-Codes: J23, J31, J51, J41, M50, D82.

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Abstract

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1 Introduction

There are two main views concerning the effect of unions on the wages and employment of nonunion workers. The first of these views (crowding or spillover view) holds that unions increase the wages of their members at the cost of reduced employment in the union sector, and thus force the displaced workers to migrate to the nonunion sector. The increase in the labor supply to the nonunion sector then makes the equilibrium wage fall. According to this view, the nonunion sector behaves competitively, and therefore unionization affects the wages and employment of nonunion workers solely through the effect that the presence of unions has on the supply of labor to the nonunion sector.

The second view (union threat view) holds, in contrast, that nonunion firms raise wages to forestall unionization. However, these firms also reduce employment in response to the higher wages, pushing the displaced workers into unemployment. Therefore, unions would benefit employed nonunion workers while hurting those who become unemployed.\(^1\)

This paper shows that, in accordance with the union threat view, workers’ ability to unionize forces nonunion employers to pay higher wages. However, in contrast to both standard views, it also shows that the threat of unionization, despite raising wages, does not lead nonunion employers to reduce their labor demand, and may even lead to an increase in employment. The explanation for this apparently puzzling result is that, when workers can unionize, employers do not behave as wage-takers, but rather consider both wages and employment as strategic variables that influence the probability of unionization. In this context, it is shown that increasing employment reduces the expected payoff to workers from unionization, so that firms have an incentive to overhire. This incentive is, in fact, strong enough to dominate the pressure to reduce employment because of the higher wages.

To analyze the effects of the threat of unionization on wages and employment, we develop a model in which firms take into account the effect of their choices on workers’ incentives to unionize. A fundamental difficulty of such a model is that the payoff workers expect to obtain from unionization, as well as the effect of firm choices on that payoff, will depend on the model used to represent union-management bargaining. To overcome this problem, we argue that, independently of the precise form of the bargaining process, it is reasonable to expect that, as

\(^1\)See Ehrenberg and Smith (1997) for a textbook exposition of both views.
employment at the outset of bargaining increases, the resulting negotiated employment will tend to increase, while the negotiated wage will tend to decrease. In other words, the outcomes of collective bargaining will tend to be monotone (nondecreasing in one case, nonincreasing in the other) in the level of employment at the outset of bargaining. We then show that the most commonly used models of union-management bargaining, the right-to-manage model (which nests the monopoly union model) and the Nash-bargaining model are monotone in the sense just described, thus proving monotonicity to be both a sensible and a general characterization of the possible bargaining processes. Therefore, to guarantee their robustness, we derive all the results for any monotone bargaining process.

The potential for the threat of unionization to increase wages was first formalized by Sherwin Rosen (1969). Even though, as mentioned above, this effect is normally associated with a reduction of employment, the possibility that the union threat may not decrease employment was already advanced by William Dickens (1986), although his insight does not appear to have been developed further. In a different context, Stole and Zwiebel (1996a) and (1996b) have also shown that, when bargaining individually with their employees, firms may have an incentive to overhire to reduce the negotiated wages. More recently, Kuhn and Padilla (2002) have obtained a similar result using a different bargaining model that allows for both individual and collective wage bargaining at the firm level.

Although our focus is different, this article is also related to a strand of research, initiated by Booth (1985), that has studied the endogenous determination of trade union membership in ‘open shop’ environments. Within this literature, Naylor and Raaum (1993) and Corneo (1995) explicitly model the role of management opposition in the determination of union membership.

The paper is organized as follows: the model is introduced in section 2; section 3 defines the concept of monotonicity of the bargaining process, and section 4 shows that the right-to-manage and the efficient Nash-bargaining models are monotone; section 5 demonstrates that it is profit-maximizing to increase wages above the reservation wage to avoid unionization, and section 6, in turn, presents the result that, despite the higher wages, employment does not fall below the

2Recently, Cahuc and Wasmer (2001) have incorporated this bargaining model into a standard matching model of the labor market.

3At an establishment with an open shop arrangement, management and a trade union negotiate about pay and other conditions of work, and the negotiated agreement extends to all employed workers, yet membership in the trade union is not compulsory.
competitive level; section 7 discusses the implications of the model and section 8 concludes.

2 The Model

Consider a firm that lives for two periods. At the beginning of the first period, the firm hires \( l_0 \) employees from a pool of identical workers, who can earn an alternative wage \( \overline{w} \) if they work elsewhere. The revenue produced by \( l_0 \) workers in the first period is \( R_0(l_0) \).

While on the job, workers accumulate firm-specific human capital. Therefore, at the end of the first period, existing employees become more valuable to the firm than alternative workers, that is, \( R(l) > \overline{R}(l) \) and \( R'(l) > \overline{R}'(l) \), where \( R(l) \) and \( \overline{R}(l) \) are the revenues the firm can obtain in the second period if it employs \( l \) skilled (i.e. with specific human capital), or unskilled workers, respectively. \( R_0, \overline{R}, \) and \( \overline{R} \) are assumed to be nondecreasing in \( l \), concave, and twice continuously differentiable functions. The acquisition of human capital also implies that \( R > R_0 \), and \( R' > R'_0 \).

At the beginning of the second period, the firm decides how many employees to retain, \( l \), and offers those workers a wage \( w \). In most models, which do not take into account the possibility that workers may organize for the purpose of collective bargaining, workers would then have two choices: they could either accept the offer or reject it, leave the firm and earn the alternative wage. Once we recognize that workers have the ability to unionize, however, there is another option open to them: they can unionize, incurring organization costs \( C(l) \). If workers unionize, collective bargaining takes place, and, as a result, \( l_u(l) \) employees are retained, and a wage \( w_u(l) \) is set. Both \( l_u \) and \( w_u \) are expressed as functions of \( l \), since the bargaining outcome may conceivably depend on the level of initial employment.

In the model, the presence of specific skills (or, more generally, of hiring, training and firing costs) creates quasi-rents over which the firm and its employees may bargain only if the latter organize to bargain collectively. The assumption that workers have no individual bargaining power in this context is made only for simplicity: none of the results in the paper would change qualitatively if we allowed for individual bargaining power, as long as employees' bargaining power is greater if they bargain collectively. \(^4\)

\(^4\)See Stole and Zwiebel (1996a) and Stole and Zwiebel (1996b) for a discussion of firm behavior under individual wage bargaining.
The model also assumes that organization is costly for workers: they need to put time and effort into organizing and may suffer from retaliatory measures that employers may take to fight the unionization attempt.\(^5\) Even if workers “hired” an existing union to set up a local at the firm, so that part of these costs would be borne by the union, they would still have to pay initiation fees and union dues. The actual process of organization, however, is left unmodeled: throughout the paper we adopt instead a reduced-form approach, summarizing the whole process by a single variable that represents the costs of unionization to workers, \(C(l)\). This formulation allows for the possibility that management opposition may defeat the unionization attempt, which would be equivalent to a high enough \(C\). In accordance with the assumption of identical workers, it is also assumed that the union is formed by all existing workers and that all workers contribute equally to the total costs of organization, so that there is a de facto closed shop.

3 Collective Bargaining

If workers unionize, the union negotiates with management over wages and, possibly, employment. The result of this negotiation will depend on the objectives of the union and the way in which bargaining takes place. Unfortunately, there is no consensus in the literature as to which is the bargaining model that best captures the actual process of union-management bargaining.\(^6\) Moreover, different models yield very different implications. For example, right-to-manage models typically imply that workers are paid their marginal products, and that the level of employment is inefficiently low. In contrast, in efficient bargaining models wages may be set above the marginal product, and the level of employment above the competitive level. In the latter models, the bargaining outcome is efficient.

However, despite this diversity, the nature of the production technology and the characteristics of the product market may impose constraints on the bargaining outcome independent of the specifics of the bargaining process. In particular, a technology with a decreasing labor marginal productivity or a downward-sloping demand curve for the firm’s output will imply that, for any given firm, the marginal revenue product of labor \(R’\) is decreasing in \(l\), as in the

\(^5\)These are very real costs. In the U.S., a measure of them is given by the number of unfair labor practice complaints issued by the National Labor Relations Board (NLRB). In 1994, the ratio of this number to the number of union recognition elections was 0.843 (Ehrenberg and Smith (1997), p. 493).

standard textbook model. And, if this is the case, one would expect that, for low levels of initial employment at the time of unionization, the union will be able to obtain wages well above the reservation wage, as infra-marginal rents per worker are large. Moreover, the union will ensure that all its members remain employed, since the benefit for the workers who remain employed from a marginal increase in the wage above the wage that guarantees full employment does not compensate for the large loss experienced by those workers who become unemployed. As initial employment increases, however, marginal workers become less productive. Therefore, the ability of the union to extract high wages while guaranteeing the full employment of its members will be weakened, and the negotiated wage will be pushed down. Finally, when membership is large enough, the terms of the trade-off between employment and wages may be reversed: at some point, workers would rather risk losing employment in exchange for a higher wage, than to secure employment at a wage just above their reservation wage.

To assess the reasonableness of this argument, it is instructive to look at the most widely used model of collective bargaining, the monopoly union model. According to this model, the union imposes its wage demands on the employer, who then sets employment at will. Therefore, the wage is always on the marginal revenue product curve. Figure 1 depicts the bargaining outcome of the monopoly union model, in which membership is determined by initial employment, $l$. In the figure, when initial employment is above $l_u^*$, the negotiated outcome is $(w_u^*, l_u^*)$. At this point, the union indifference curve ($IC^*$) is tangent to the marginal revenue product schedule ($R'$). For example, if the level of initial employment is equal to $l_c$ (which would be the competitive employment level), the union is willing to accept some unemployment for its members in exchange for a high wage paid to employed union workers. For levels of initial employment below $l_u^*$, such as $l'$, the union does not want to give up employment in exchange for a higher wage, and, thus, $l_u(l') = l'$. Note also that the firm’s profits increase as the wage falls and employment increases along the marginal revenue product curve. In the picture, the firm’s isoprofit curve at $(w_u(l'), l_u(l'))$, ($IP'$), is above the isoprofit curve at $(w_u^*, l_u^*)$, ($IP^*$). Since profits increase as we move to the southeast, this implies that profits are higher at $(w_u^*, l_u^*)$.

We will use the term monotone to designate any bargaining process that exhibits the relationship between membership and the negotiated outcome just described. The term has been chosen to capture the idea that negotiated employment is monotonically nondecreasing and the
negotiated wage is monotonically nonincreasing in initial employment. The following definition formalizes the argument.

**Definition 1 (Monotonicity)** A bargaining process is monotone in $l$ if:

(i) $w_u(l)$ is nonincreasing in $l$.

(ii) There exists a level of membership $l_u^*$ such that:

   a. For $l \leq l_u^*$, $l_u(l) = l$

   b. For $l \geq l_u^*$, $l_u(l) = l_u(l_u^*)$ and $w_u(l) = w_u(l_u^*)$.

(iii) $\Pi_U(l)$ is nondecreasing in $l$,

where $\Pi_U(l)$ are the profits earned by a unionized firm that was employing $l$ workers before unionization.

Condition (i) states that a larger membership will induce the union to accept a lower wage in exchange for greater employment. Condition (ii) reflects the idea that, for low levels of initial employment, the negotiated outcome implies the employment of all union members (ii.a), while for high enough levels of initial employment, the union is not willing to concede any further reduction in wages and instead accepts some unemployment (ii.b). Condition (ii) thus implies
that $l_u(l)$ is nondecreasing in $l$. Note that condition (ii.b) can also be interpreted as requiring a bargaining problem with no restrictions on $l_u$ to have a solution. This interpretation shows that assumption ii.b. is always made implicitly in the literature. Finally, condition (iii) means that, as in the monopoly union model in Figure 1, restricting initial employment below $l_u^*$ does not increase the firm’s profits.

Although no formal derivation of the results was offered, Figure 1 suggests that the monopoly union model is monotone. The next propositions show that not only the monopoly union model, but also the two other most commonly used models of union-management bargaining are monotone: the right-to-manage model (of which the monopoly union model above is a special case) and the efficient bargaining model.

4 Monotone Bargaining Processes

4.1 Union Objectives

As mentioned above, there is no consensus as to which is the most adequate choice of the union’s objective function (and even to whether union decisions can be represented at all as the outcome of some form of maximization akin to individual utility maximization). In the following sections, we will assume that the union maximizes the expected utility of all its members, and that workers are risk-neutral, so that the union strives to maximize its members’ total rents. The union’s objective function, $U$, is thus:

$$U(l_u, w_u; l, w) \equiv l_u(w_u - w)$$

The choice of this specific function, which is probably the single most used objective function in applications, does not appear to be especially restrictive, as it can be easily extended to accommodate different union objectives. For example, nothing would change if the union cared only about initial members, since the negotiated employment cannot be greater than initial employment. In models where members are heterogeneous, the union may maximize the expected utility of only some class of workers (like in insider-outsider models) or the expected utility of the median-voter; in dynamic models, the union would care about some appropriately discounted sum of the future expected utility of its members, and so on. In all cases, the union’s objectives can be represented by a function like $U$. Moreover, it can be shown that many of the
results derived below would hold for more general objective functions.\footnote{For example, the results derived for the right-to-manage model would also hold if we instead used a concave function of the form $\hat{U}(l_u, w_u, \overline{w}) = g(l_u, \overline{w})h(w_u, \overline{w})m(l, \overline{w})$. This formulation nests the expected-utility function with risk-averse or risk-neutral workers, and the Stone-Geary function, commonly used in empirical work.}

Since $\overline{w}$ will be assumed to be a constant throughout the paper, $U(l_u, w_u; \overline{w})$ will simply be written $U(l_u, w_u)$.

\section*{4.2 Bargaining Models}

\subsection*{Right-to-Manage Models}

Right-to-manage models assume that the firm and the union bargain over the wage, but management keeps the prerogative to set employment at will. These models are motivated by the fact that employment is rarely a subject of collective bargaining agreements, at least not in an explicit way. In most applications, it is assumed that the negotiated wage can be described as the generalized Nash-bargaining outcome of a bargaining problem in which the effect of wages in employment is taken into account. Formally, given $l$, $w_u(l)$ is the solution of the following maximization problem:

$$\max_{w_u} \quad F(w_u, l) = \phi \ln(U(l_u(w_u, l), w_u)) \quad (RTM)$$

$$+ (1 - \phi) \ln(R(l_u(w_u, l)) - w_u l_u(w_u, l)),$$

where $l_u(w_u, l)$ is the solution of management’s employment choice problem $(L)$ given $w_u$ and $l$:

$$\max_{l_u} \{ R(l_u) - w_u l_u \} \quad (L)$$

$$s.t. \ l_u \leq l,$$

and $\phi$ represents the union’s “bargaining power”. It is assumed, with most of the literature, that the disagreement payoffs for the union and the firm are both zero. That is, in case of disagreement, workers can earn $\overline{w}$, and the firm does not operate.\footnote{If the Nash-bargaining solution is justified as an approximation to noncooperative bargaining solutions, the disagreement payoffs should generally be those that the parties would obtain in case of “perpetual disagreement”. One could think that these payoffs could be different from zero, if the firm can use replacement workers, or if union workers can earn less than $\overline{w}$ while negotiations proceed. None of the results would change in any qualitative way by assuming different disagreement payoffs. See Muthoo (1999) for a discussion of the Nash-bargaining outcome and its relation to noncooperative bargaining solutions.}

The next proposition shows that, as hinted by the discussion of the monopoly union model above, right-to-manage models are monotone (all omitted proofs can be found in the appendix).
Proposition 1  Let $R'$ be a concave function. Then, the right-to-manage model is monotone.

Efficient-Bargaining Models

In contrast to the inefficient right-to-manage outcome, efficient bargaining models assume that management and the union strike an efficient agreement. To obtain a unique efficient solution, it is commonly assumed that the bargaining outcome can be characterized by the generalized Nash-bargaining solution, that is, the solution to the following problem:

$$\max_{l_u, w_u} \phi ln(U(l_u, w_u)) + (1 - \phi)ln(R(l_u) - w_ul_u)$$

s.t. $l - l_u \geq 0$. (NB)

The following proposition shows that, in this case, the bargaining process is also monotone.

Proposition 2  The generalized Nash-bargaining model with joint bargaining over wages and employment is monotone.

Propositions 1 and 2 thus confirm the appeal of monotonicity: not only does it seem to be a sensible feature of a bargaining process, but it also characterizes the most widely used bargaining models. Therefore, in the following sections we will abstract from the specificities of the bargaining process and simply assume that it is monotone. All results will automatically apply to the bargaining models discussed in this section.

5  Wage Setting and Union Avoidance

In the second period, the firm has to offer a wage at least as high as workers’ expected payoff from unionization (net of organization costs) if it wants to remain nonunion. The following proposition shows that remaining nonunion is the firm’s optimal strategy.\textsuperscript{10}

\textsuperscript{9}This is just a technical assumption made to guarantee that the solution to problem (RTM) can be characterized by its first-order condition. The concavity of $F$ is customarily, yet often implicitly, made in the literature, and dropping it would make the solution to (RTM) much more cumbersome without significantly altering the results.

\textsuperscript{10}The result generalizes to the case in which workers are risk-averse.
Proposition 3  If the bargaining process is monotone, then, at the unique subgame perfect equilibrium of the second period subgame, the firm offers

\[ w(l^*) = \max \{ w^T(l^*), \bar{w} \}, \]

where

\[ w^T(l) \equiv \bar{w} + \frac{l_u(l)}{l} (w_u(l) - \bar{w}) - \frac{C(l)}{l}, \]

and \( l^* \) is the optimal employment level, and workers accept the wage offer.

It is straightforward to see why it is optimal for the firm to avoid unionization. For \( l \) such that \( l_u(l) = l \), unionization would not reduce revenues, but it would imply a larger wage bill \((w_u l)\) than paying workers their expected payoff from unionization \((w_u l - C(l))\). For \( l \) such that \( l_u(l) < l \), unionization would reduce revenues, while its effect on the wage bill would depend on the magnitudes of \((l - l_u)\bar{w}\) and \(C(l)\). While the latter effect could make unionization beneficial to the firm for sufficiently large values of \( l \), Proposition 3 shows that, at the optimal \( l \), it is best for the firm to avoid unionization.

Therefore, as proposed by the union threat view described in the introduction, in equilibrium, firms raise wages above \( \bar{w} \) to completely eliminate workers’ incentives to organize, and no unionization takes place. Only if \( C(l) \) is sufficiently large will the union threat become ineffective, and the wage remain at the competitive level.

Note that we have assumed that the costs of organization are borne equally by all workers, and that the firm tries to convince all workers not to unionize. Introducing worker heterogeneity with respect to the costs of organization or allowing the firm to discriminate when setting wages would not alter the results in any qualitative way.\(^{11}\)

6 Employment Determination

Proposition 3 shows that it is profit-maximizing for the firm to pay a wage above \( \bar{w} \) in the second period to ensure that workers do not unionize. But it also shows that the wage necessary

\(^{11}\)It is not clear whether the firm would be able to pay different wages to workers carrying out the same tasks, and, interestingly, whether, if possible, it would be optimal to do so. In fact, apart from the potentially disruptive morale problems that could ensue, Dickens (1986) has shown that, under certain conditions, it is cheaper to pay all workers the same wage to dissuade them from organizing.
to forestall unionization is influenced by second-period employment, $l$. Therefore, the firm will take this influence into account when choosing its optimal employment level.

Before analyzing how this influence plays out, note that, when choosing second-period employment, $l$, the firm is constrained by the first-period choice of employment, $l_0$. The determination of employment is thus an intertemporal decision problem, in which the choices of first- and second-period employment are interrelated. However, it is instructive to leave aside the intertemporal nature of the firm’s problem and look first at what the firm would do in the second period if there were no constraints on $l$. The following proposition shows that the firm’s optimal employment choice in such case, $l_T$, is greater than the competitive employment level $l_c$.

**Proposition 4** Let $l_T$ be the solution to

$$
\max_l R(l) - w(l)l \quad s.t. \quad w(l) = \frac{l_u(l)}{l_u(l)}(w_u(l) - \bar{w}) + \bar{w} - \frac{C(l)}{l}
$$

Then, $l_T \geq l_c$.

Proposition 4 is a surprising result: it implies that the union threat leads firms to increase both the wage and employment above competitive levels. However, it is straightforward to see how it follows immediately from the monotonicity of the bargaining process. First, monotonicity implies that, for $l < l_u^*$, the profits of a nonunion firm are equal, leaving aside costs of organization, to those that would be obtained if workers unionized (as, in this case, $l_u(l) = l$). Therefore, it pays off for the firm to employ at least $l_u^*$ workers, since monotonicity (condition (iii)) implies that a lower employment level would reduce the firm’s profits under the union. Moreover, for $l \geq l_u^*$, monotonicity implies that employing additional workers would not alter the union outcome. To see what this implies for the firm’s employment choice, it is illuminating to imagine what would happen if hiring and firing took place sequentially, so that the last workers hired were the first fired. In this case, once $l_u^*$ workers have been hired, the next worker hired knows that he will be fired in case of unionization. Therefore, it would be enough to pay this worker $\bar{w}$ to convince him not to unionize, as he would not have anything to gain from unionization. This implies that, as long as $C$ is nondecreasing in $l$, the marginal cost of employment for $l > l_u^*$ is actually lower than $\bar{w}$, which explains the overemployment result. Although firing is random in the model, the same intuition applies.
A different way to understand the result is to realize that, when workers can unionize, the marginal cost of labor is not constant and equal to the wage, as in competitive models. Rather, since increasing $l$ reduces workers’ expected payoff from unionization (either by reducing the expected union wage or the probability of obtaining it), the marginal cost of labor is both lower than the wage, so that the optimal employment-wage pair lies above the labor demand curve, and decreasing in $l$.

It is worth mentioning that the wage-employment pair that results from the firm’s maximization problem is above the marginal revenue product curve (the competitive labor demand curve). This type of outcome has been criticized in the context of wage bargaining models on the grounds that it may not be incentive-compatible. The reasoning behind this criticism is that the firm has incentives to reduce its labor force after the agreement has been reached, since, at the negotiated outcome, the marginal cost of labor is higher than its marginal revenue product. In our case, however, the wage-employment pair is incentive-compatible for the firm unless the threat of unionization is a one-time event: as long as workers have the ability to unionize, it is profit-maximizing for the firm not to deviate from the wage and employment levels derived in Proposition 4.

As it was noted in the introduction, the possibility that the threat of unionization may lead to over-employment had already been explored by Dickens (1986). Using a different modeling strategy and different assumptions about the ability of the union to discriminate between workers with respect to wages and employment, Dickens (1986) showed that the threat of unionization could increase or decrease employment. He realized that, for a given bargaining outcome, increasing initial employment reduces the expected rents per worker of unionization, although he did not seem to realize that the level of initial employment affects the negotiated outcome itself. This section extends and generalizes these results, showing that, for a large class of bargaining models, which includes the most commonly used models of union-management bargaining, no underemployment will result.

It is also worth mentioning that a result along similar lines has been obtained by Stole and Zwiebel (1996a) and (1996b) for the case of individual worker-firm bargaining. In these two papers, Stole and Zwiebel showed that, for a firm bargaining individually with its workers, it is

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12 In particular, Dickens assumed that, if workers unionize, those who were active in the unionization process can ensure that they are laid off only after all workers who were not active.
optimal to hire more workers than in the competitive case, because, by doing so, the firm can strike a better wage deal with all of them. Kuhn and Padilla (2002) obtained a similar result using a different bargaining model. In their model, when workers bargain individually, overhiring ensues for the same reasons as in Stole and Zwiebel (1996a) and (1996b). When workers bargain collectively, Kuhn and Padilla (2002) also obtained an overemployment result because workers competing for jobs at unionized firms underbid the market wage. It should be emphasized, however, that, in these papers, there is some sort of wage bargaining between workers and the firm, while, in our model, the firm unilaterally sets wages. The threat of collective bargaining, not the actual bargaining between workers and the firm, is what drives our overhiring outcome.

There are two important dimensions of the firm’s problem, however, that are neglected by the one-period result in Proposition 4. First, the firm may be able to undo the effects of the union threat if it can recover in the first period, by setting a low enough \( w_0 \), the rents that it will be forced to pay workers in the second period. And second, workers’ lower productivity in the first period may lead firms to set \( l_0 \) below \( l_T \), which may attenuate the overemployment result in Proposition 4. The intertemporal problem is fully analyzed next.

### 6.1 Intertemporal Wage and Employment Determination

To determine initial employment, \( l_0 \), and wages, \( w_0 \), and second-period employment, \( l \), the employer solves the following problem:

\[
\begin{align*}
\max_{w_0, l_0, l} & \quad R_0(l_0) - w_0 l_0 + \delta [R(l) - w(l) l] \\
\text{s.t.} & \quad w(l) = \max \{ w^T(l), \bar{w} \} \\
& \quad w_0 + \delta \left[ \frac{l}{l_0} w(l) + \frac{l_0 - l}{l_0} \bar{w} \right] \geq \bar{w}(1 + \delta) \\
& \quad l \leq l_0,
\end{align*}
\]

(Emp)

where

\[
w^T(l) \equiv \frac{l_u(l)}{l} (w_u(l) - \bar{w}) + \bar{w} - \frac{C(l)}{l}.
\]

The parameter \( \delta > 0 \) captures both the usual time-discount factor (assumed equal for firms and workers) and the fact that the period during which workers accumulate specific skills (the first period in the model) may be of different duration than the period during which skilled
workers remain in the firm (the model’s second period). If both periods were of equal length, then $\delta$ would be a pure time-discount factor and, thus, would be less than one. However, if, as it is more likely, the expected tenure in the firm is large relative to the apprenticeship period, $\delta$ may be larger than one.

Constraint (1) ensures that unionization does not take place in the second period (and also that workers want to remain in the firm), while (2) is the first-period participation constraint. At the optimum, (2) will be binding, so that

$$w_0 = \overline{w} - \delta \frac{l}{l_0}(w(l) - \overline{w}) \quad (4)$$

Therefore, substituting (4) into the firm’s profits, the employer’s problem reduces to (COMP):

$$\max_{l_0, l} \quad R_0(l_0) - \overline{w}l_0 + \delta[R(l) - \overline{w}l] \quad \text{(COMP)}$$

subject to

$$l \leq l_0,$$

which is identical to the problem of a firm faced with a perfectly competitive labor market. Therefore, the existence of a union threat would not affect the firm’s employment choices.

**No-commitment.** The above conclusion is hardly surprising, since, on the one hand, the firm is able to recover, through a lower $w_0$, the rents that it needs to grant workers in the second period to avoid unionization (by setting its first-period wage according to (4)), and, on the other hand, it can commit to the announcement of $l$ made in the first period. Therefore, we have sidestepped the potential distortions that the existence of quasi-rents in the second period may create. If a firm can deviate from its announced $l$, however, it is conceivable that, since $w_0$ depends on $l$, it may announce a level of second-period employment different from the one it will choose in the second period, if, by doing so, it lowers $w_0$. If workers foresee this, the firm’s problem departs from (EMP), since the firm’s choice of $l$ needs to be incentive-compatible in the second period, that is, $l$ needs to maximize second-period profits, given the choice of $l_0$:

$$R(l) - w(l)l \geq R(l') - w(l')l', \text{ for all } l' \leq l_0 \quad \text{(IC)}$$

\textsuperscript{13}If workers could find employment in other firms where they could potentially unionize, the relevant reservation payoff would be the maximum of their expected net present payoff in those firms and the right-hand-side of inequality (2). Note that, if the supply of workers were infinitely elastic at $\overline{w}$, in equilibrium, (2) would be binding in any case.

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The next proposition shows, however, that the threat of unionization does not affect employment choices, even if the firm cannot credibly commit to second-period employment.

**Proposition 5** Let \((\hat{l}_0, \hat{l})\) be the solution to the two-period competitive problem \((COMP)\), and \((l^*_0, l^*)\) the solution to the firm’s problem under the threat of unionization \((P)\):

\[
\max_{w_0,l_0,l} R_0(l_0) - w_0l_0 + \delta[R(l) - w(l)l]
\]

s.t. \((1) - (3)\), and \((IC)\).

Then, \(l^*_0 = l^* = \hat{l}_0 = \hat{l}\).

The reason why the addition of constraint \((IC)\) does not alter the result obtained under commitment is that, as Proposition 4 shows, the union threat provides incentives to hire more rather than less workers. Therefore, if a firm announces to hire the competitive level of employment, the union threat will certainly not lead it to reduce employment below the announced level. Moreover, the fact that workers are less productive in the first period implies that, at the competitive solution, \(\hat{l}_0 = \hat{l}\). Therefore, in the second period the firm cannot deviate either from the competitive level by increasing \(l\) above \(\hat{l}\), as it is constrained by \(\hat{l}_0\).

### 6.2 Liquidity-Constrained Workers and Minimum Wages

The irrelevance of the union threat for employment implied by Proposition 5 hinges on the ability of employers to pass on to workers, in the form of lower initial wages, the additional second-period wage costs caused by the threat of unionization. However, this ability is likely to be, at best, limited in actual markets. To see this, notice that, if a worker expects to remain in the firm for long after acquiring specific skills, the parameter \(\delta\) may be much larger than one, and, thus, the minimum wage that workers are willing to accept to join the firm \((w_0 = \bar{w} - \delta \hat{l}_0(w(l) - \bar{w}))\) may be very low. If that is the case, liquidity constraints on the part of workers, or minimum wage regulations may imply that such a \(w_0\) is not feasible. Therefore, it is necessary to study the firm’s wage and employment determination problem when there is a lower bound for \(w_0\).

This lower bound is incorporated in the model by requiring \(w_0\) to be above an exogenously given
minimum \( w\):

\[
w_0 \geq \underline{w}
\]  

(LB)

The next proposition shows that constraining the ability of the firm to pass on second-period labor costs to workers through low first-period wages does not lead to lower employment: even if the union threat unequivocally increases the total discounted cost of labor, so that the firm is forced to share rents with workers, employment will not be reduced relative to the competitive problem.

**Proposition 6** Let \( C \) be a convex function,\(^{15}\) and \((\tilde{l}_0^*, \tilde{l}^*)\) be the solution to \((P_R)\):

\[
\max_{w_0, l_0, l} R_0(l_0) - w_0 l_0 + \delta[R(l) - w(l)l]
\]

s.t. \((1) - (3), \ (IC), \ and \ (LB)\).

Then \( \tilde{l}_0^* \geq \hat{l}_0 \) and \( \tilde{l}^* \geq \hat{l} \).

In fact, if \( w \) and \( \delta \) are sufficiently large, the firm’s problem will approach the one-period problem discussed at the beginning of this section. Therefore, as Proposition 4 shows, if the firm cannot recover the rents shared with workers in the second period, employment will tend to be higher than in the competitive case.

7 Discussion

The previous sections have analyzed the behavior of a single firm. If we consider an industry composed of identical firms facing an infinitely elastic labor supply at wage \( \overline{w} \),\(^{16}\) the results in propositions 4-6 extend to the industry’s labor market equilibrium. Summarizing these results, the partial equilibrium implications of workers’ ability to unionize are as follows:

1. The wage paid to experienced workers (those with the ability to unionize) will be higher than in the competitive case, and it will be increasing in the size of the quasi-rents and the expected bargaining power of the union, and decreasing in organization costs.

---

\(^{15}\)Convexity of \( C \) is not a necessary condition. For the result to go through we only need \( \frac{C(l)}{l} \) not to decrease too fast with \( l \).

\(^{16}\)The wage \( \overline{w} \) may be considered as the value of leisure or self-employment if this is the only industry in the economy, or the wage paid in an alternative industry.
2. The wages of unexperienced workers will be lower than in the competitive case, and they will be negatively related to the wages of experienced workers.

3. The wage-tenure profile will be steeper than in the competitive case. This implication follows directly from the previous two.

These three implications are shared with models in which firm-specific human capital creates rents over which firms and employees bargain individually. The present model, however, shows that rent-sharing can take place even if there is no actual bargaining, and even if workers have no individual bargaining power. All that is required is that there exists a threat of unionization. Moreover, the model has the additional implication that both the wage levels and their difference will depend on the costs of organization and the union’s bargaining power.

4. The net present value of wages will be equal or greater than in the competitive case. This immediately implies that, if there is time-discounting, or if the constraint on $w_0$ is binding, the average wage (averaging across periods) will be higher than in the competitive case.\(^\text{17}\) In the latter case, the net present value of wages will also be strictly higher than in the competitive case.

5. The levels of employment of both experienced and unexperienced workers will be equal or greater than the competitive employment levels.

Therefore, in a partial equilibrium context the implications of the model depart from those of the textbook union threat view, since the increase in wages caused by the union threat is not accompanied by a reduction in employment. The comparison with the crowding view is less straightforward, because, in the model, no unions form in equilibrium. However, even if they did for some reason not specified in the model, and even if their presence reduced $\bar{w}$, the net effect on nonunion wages could be positive or negative, depending on the relative strengths of the supply shift (the reduction in $\bar{w}$) and the upward pressure on wages caused by the union threat. In any case, even if wages fell as a consequence of unionization, the decrease would be smaller than the one that would follow from a purely competitive model.

An additional implication that follows from monotonicity is that the hiring activity of unionized employers may increase significantly right before contract renegotiation, as firms try to put pressure on unions to accept lower wages.

\(^{17}\)Time-discounting implies that the present value of a sequence of future wages is less than the sum of that sequence.
8 Conclusion

A basic feature of the labor markets of most advanced economies is that workers can—or at least can try to—organize to bargain collectively with their employers. This possibility implies that employers will take into account the effect of their choices on workers’ incentives to unionize and thus adds a strategic dimension to wage and employment determination. This paper has shown that, once we consider the strategic nature of wage and employment determination under the threat of unionization, we obtain predictions that differ from the ones derived from standard models. In particular, the main result of this paper is that workers’ ability to unionize will lead nonunion firms to pay higher than competitive wages and, at the same time, to set a level of employment equal or greater than the competitive employment level. We show this result to hold for a very general class of potential union-management bargaining processes, including the most commonly used ones in the literature.
9 Appendix

Proof of Proposition 1. 1. Solution to the unconstrained problem. Let \( w_u^* \) be the solution to problem (A):

\[
\max_{w_u} F_A(w_u), \tag{A}
\]

where

\[
F_A(w_u) \equiv \phi \ln(U(l_{uA}(w_u), w_u) + (1 - \phi) \ln(R(l_{uA}(w_u)) - w_u l_{uA}(w_u)), \tag{5}
\]

and \( l_{uA}(w_u) \equiv \arg \max_{l_u} \{R(l_u) - w_u l_u\} \). That is, problem (A) is a bargaining problem with no constraints on \( l_u \). Given concavity of \( R \), \( l_{uA}(w_u) \) satisfies

\[
R'(l_{uA}^*(w_u)) = w_u \tag{6}
\]

\[
l'_{uA}(w_u) = \frac{1}{R''(l_{uA}(w_u))} < 0 \tag{7}
\]

The first order condition of problem (A) is:

\[
\phi \frac{1}{w_u^* - \overline{w}} + \phi \frac{l'_{uA}(w_u^*)}{l_{uA}(w_u^*)} - (1 - \phi) \frac{l_{uA}(w_u^*)}{R(l_{uA}(w_u^*)) - w_u^* l_{uA}(w_u^*)} = 0
\]

It can be checked that \( R''' \leq 0 \) is a sufficient condition for the concavity of \( F_A \) and, therefore, also a sufficient condition for the maximality of \( w_u^* \).

Let \( l_u^* = l_{uA}(w_u^*) \), and let \( l_c = l_{uA}(\overline{w}) \) be the competitive employment level. Then, by concavity of \( R \), \( w_u^* > \overline{w} \) implies that \( l_u^* < l_c \).

2. Solution to (RTM) when \( l > l_u^* \). Define \( l_u(w_u, l) \) as the solution to (L):

\[
\max_{l_u} \{R(l_u) - w_u l_u\} \tag{L}
\]

s.t. \( l_u \leq l \).

For \( l > l_u^* \), we know from problem (A) that \( F(w_u^*, l) = F_A(w_u^*) > F_A(w_u) \) for any \( w_u < w_u^* \). That is, if \( l_u(w_u, l) = l_{uA}(w_u) \), then \( F(w_u^*, l) > F(w_u, l) \), or, put differently, \( (w_u^*, l_u^*) \) is preferred to any other \( (w_u, l_u) \) on the \( R' \) curve. However, the restriction \( l_u \leq l \) in problem (L) implies that bargaining outcomes with \( w_u < R'(l) \) and \( l_u(w_u, l) = l < l_{uA}(w_u) \) could be possible. We need to check that \( (w_u^*, l_u^*) \) is also preferred to any \( (w_u, l_u) \) such that \( w_u < R'(l) \).
Let $w_u < R'(l)$, so that the restriction in problem (L) is strictly binding. In this case:

$$F_{w_u}(w_u, l) = \frac{1}{w_u - w} - (1 - \phi) \frac{l}{R(l) - w_u l},$$

and

$$F_{w_u w_u} = -\phi \frac{1}{(w_u - w)^2} - (1 - \phi) \frac{l^2}{(R(l) - w_u l)^2} < 0,$$

so that, for $w_u < R'(l)$, $F_{w_u}(w_u, l) > F_{w_u - (R'(l), l)}$, and

$$F_{w_u - (R'(l), l)} = \phi \frac{1}{R'(l) - w_u} - (1 - \phi) \frac{l}{R(l) - R'(l) l}$$

$$> \phi \frac{1}{R'(l) - w_u} + \phi \frac{\phi}{w_u R'(l)} l - (1 - \phi) \frac{l}{R(l) - R'(l) l} = F'_A(R'(l)) > 0,$$

where the last inequality follows from $F'_A(w_u^*) = 0$, $R'(l) < w_u^*$, and concavity of $F_A$.

Therefore, $F_{w_u}(w_u, l) > 0$ for $w_u < R'(l)$, which implies that $(l, R'(l))$ is preferred to $(l, w_u)$ for $w_u < R'(l)$, and thus that $(w_u^*, l_u^*)$ is preferred to any $(w_u, l)$ with $w_u \leq R'(l)$. It follows that for any $l > l_u^*$, $(w_u^*(l), l_u(l)) = (w_u^*, l_u^*)$, so that condition (ii.b) in the definition of monotonicity is satisfied.

3. Solution to (RTM) when $l \leq l_u^*$. For $l < l_u^*$, if $w_u > R'(l)$, $l_u(w_u, l) = l_u A(w_u)$, so that $F(w_u, l) = F_A(w_u)$. It follows that, for $w_u > R'(l)$, $F_{w_u}(w_u, l) = F'_A(w_u) < 0$ and, therefore, that $w_u(l) \leq R'(l)$ and $l_u(l) = l$, so that condition (ii.a) in the definition of monotonicity is satisfied.

Now, let $H(l)$ be defined as:

$$H(l) = \phi \frac{1}{R'(l) - w_u} - (1 - \phi) \frac{l}{R(l) - R'(l) l},$$

that is, $H(l) = F_{w_u - (R'(l), l)}$. Since, we showed above that $F_{w_u - (R'(l), l)} > F_{w_u}(w_u, l)$, for any $w_u < R'(l)$, $H(l) \geq 0$ implies that $w_u(l) = R'(l)$, and $H(l) < 0$ implies that $w_u(l) < R'(l)$. For $H$ so defined:

$$H'(l) = -\phi \frac{R''(l)}{(R'(l) - w_u)^2} - (1 - \phi) \frac{R - R' + l^2 R''}{(R - R')^2}$$

Concavity of $R'$ implies that $-lR''(l) \geq R'(0) - R'(l)$, so that:

$$R(l) - R'(l) l + l^2 R''(l) \leq R(l) - R'(l) l + l(R'(l) - R'(0)) = R(l) - lR'(0) < 0,$$
where the last inequality follows from concavity of $R$. Therefore $H'(l) > 0$.

Now, recall that $F_A'(w_u^*) = 0$, which, since $l_u' < 0$ and $w_u^* = R'(l_u^*)$, implies

$$H(l_u^*) = F_A'(w_u^*) - \frac{F_A''(w_u^*)}{l_u^*} > 0$$

Therefore, since $H' > 0$, there exists an $l_b < l_u^*$ such that, for $l \geq l_b$, $H(l) \geq 0$ and $w_u(l) = R'(l)$, and, for $l < l_b$, $H(l) < 0$ and $w_u(l) < R'(l)$. Hence, for $l \geq l_b$, $w_u(l)$ is decreasing in $l$ because of concavity of $R$. For $l < l_b$, $w_u(l)$ is given by:\[18\]

$$
\phi \frac{1}{w_u(l) - \overline{w}} - (1 - \phi) \frac{l}{R(l) - w_u(l)} = 0,
$$

or, rearranging,

$$w_u(l) = \phi \frac{R(l)}{l} + (1 - \phi)\overline{w}
$$

Therefore $w_u(l)$ is also decreasing in $l$ for $l < l_b$, so that $w_u(l)$ is decreasing for $l < l_u^*$. Together with condition (ii.b), this implies condition (i).

Condition (iii) follows immediately from the above argument. For $l < l_b$, $w_u(l) = \phi \frac{R(l)}{l} + (1 - \phi)\overline{w}$, so that $\Pi_U(l) = (R(l) - \overline{w})(1 - \phi)$ and $\Pi_U'(l) = (R'(l) - \overline{w})(1 - \phi) \geq 0$. For $l \in (l_b, l_u^*)$, $w_u(l) = R'(l)$, so that $\Pi_U'(l) = R'(l) - R'(l) - R''(l)l \geq 0$.

**Proof of Proposition 2.** The generalized Nash-bargaining problem is:

$$\max_{l_u, w_u} \phi \ln(U(l_u, w_u)) + (1 - \phi)\ln(R(l_u) - w_u l_u) \quad \text{(NB)}$$

$$\text{s.t. } l - l_u \geq 0$$

Given that concavity of $R$ guarantees the concavity of the maximand, the necessary and sufficient conditions for a maximum are (after some rearranging):

$$w_u = \phi \frac{R}{l_u} + (1 - \phi)\overline{w}
$$

$$w_u = \phi \frac{R}{l_u} + (1 - \phi)R' - \lambda(R - w_u l_u)
$$

$$\lambda(l - l_u) = 0
$$

$$\lambda \geq 0
$$

$$l - l_u \geq 0
$$

\[18\]Note that $l_b$ could be negative, so that there could be no $l \geq 0$ such that $H(l) < 0$. Note also that $F_{w_u} \to \infty$ as $w_u \to \overline{w}$, so that, for $l < l_b$, at the maximum $F_{w_u} = 0$. 

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Now, note that concavity of $R$ guarantees that there is a unique optimum, $(w_u^*, l_u^*)$, to a modified problem in which the restriction $l - l_u \geq 0$ is eliminated (i.e. $(w_u^*, l_u^*)$ solves the above system of equations for $\lambda = 0$). It is a standard result in the literature (see, for example, Booth (1995)) that $l_u^* = l_c$, where $l_c$ is the competitive outcome.

If (17) is not binding, $\lambda = 0$, and the solution $(l_u(l), w_u(l))$ is identical with $(w_u^*, l_u^*)$. Therefore, for $l \geq l_u^*$, $(w_u(l), l_u(l)) = (w_u^*, l_u^*)$, so that condition (ii.b) of the definition of monotonicity is satisfied.

If $l < l_u^*$, it has to be the case that $\lambda > 0$ (as $(w_u^*, l_u^*)$ is the unique solution to (13)-(17) if $\lambda = 0$), so that (17) will be binding. It follows that $l_u(l) = l$, satisfying condition (ii.a), and that $w_u(l)$ is given by (13), so that $w_u(l)$ is decreasing in $l$ for $l < l_u^*$. Together with condition (ii.b) this implies that condition (i) is satisfied.

Now, for $l < l_u^*$, \( \frac{d(w_u(l) l_u(l))}{dl} = \phi R'(l) + (1 - \phi)w \), so that $\Pi_U(l)' = R'(l) - \frac{d(w_u(l) l_u(l))}{dl} = (1 - \phi)(R'(l) - w) > 0$, since $l < l_u^* = l_c$, which proves condition (iii). \[\blacksquare\]

**Proof of Proposition 3.** If the firm offers a wage below $w^*(l)$, workers form a union and profits are $\Pi_U(l) = R(l_u(l)) - w_u(l)l_u(l)$. If it offers $w^*(l)$, and workers accept, profits are:

\[
\Pi(l) = R(l) - l \left( \frac{l_u(l)}{l} w_u(l) + \frac{l - l_u(l)}{l} \bar{w} - \frac{C(l)}{l} \right) = R(l) - l_u(l) w_u(l) - \bar{w}(l - l_u(l)) + C(l)
\]

(18)

Let $l^*_u$ be as described in the definition of monotonicity and let $l^*$ be the optimal employment level given the constraint $l \leq l_0$. It follows that $\Pi_U(l^*_u) \geq \Pi_U(l)$ for any $l$. Moreover, $\Pi(l^*_u) = R(l^*_u) - l^*_u w^*u + C(l^*_u) > \Pi_U(l^*_u)$. It follows that, if $l^*_u$ is feasible ($l_0 \geq l^*_u$), $\Pi(l^*) \geq \Pi(l^*_u) > \Pi_U(l)$ for any feasible $l$. If $l^*_u$ is not feasible ($l_0 < l^*_u$), then, by monotonicity, $l_u(l) = l$, so that, for any feasible $l$, $\Pi(l) = R(l) - l_u(l) w_u(l) + C(l) = \Pi_U(l) + C(l)$ Therefore, $\Pi(l^*) > \Pi_U(l)$ for any feasible $l$. \[\blacksquare\]

**Proof of Proposition 4.** For $l < l_u^*$, monotonicity implies that $l_u(l) = l$, so that $w(l) = w_u(l) - \frac{C(l)}{l}$, and profits are $\Pi(l) = R(l) - w_u(l)l + C(l) = \Pi_U(l) + C(l)$. Monotonicity also implies that $\Pi_U(l)$ is nondecreasing in $l$. Therefore, if $C$ is nondecreasing, for $l < l_u^*$, $\Pi(l)$ is nondecreasing in $l$, so that $l_T \geq l_u^*$.

Now, for $l > l_u^*$, monotonicity implies that $l_u(l) = l_u^*$, and $w_u(l) = w_u^*$, so that $w(l)l = \Pi_U(l) = R(l) - l_u^* w_u^* + C(l) = \Pi_U(l) + C(l)$. Monotonicity also implies that $\Pi_U(l)$ is nondecreasing in $l$. Therefore, if $C$ is nondecreasing, for $l < l_u^*$, $\Pi(l)$ is nondecreasing in $l$, so that $l_T \geq l_u^*$.

Now, for $l > l_u^*$, monotonicity implies that $l_u(l) = l_u^*$, and $w_u(l) = w_u^*$, so that $w(l)l = \Pi_U(l) = R(l) - l_u^* w_u^* + C(l) = \Pi_U(l) + C(l)$. Monotonicity also implies that $\Pi_U(l)$ is nondecreasing in $l$. Therefore, if $C$ is nondecreasing, for $l < l_u^*$, $\Pi(l)$ is nondecreasing in $l$, so that $l_T \geq l_u^*$.
Proposition 5. First note that the assumption that \( R'(l) > R'_0(l) \) for all \( l \) implies that the restriction \( l \leq l_0 \) is binding at the solution to the competitive problem \((COMP)\), since 
\[
R'_0(l_0) - \overline{w} + \delta(R'(l_0) - \overline{w}) < 0.
\]
Therefore, the competitive solution is \((\hat{l}, \hat{l})\), where \( \hat{l} < l_0 \) satisfies:
\[
R'_0(\hat{l}) - \overline{w} + \delta(R'(\hat{l}) - \overline{w}) = 0.
\]
It follows that \((\hat{l}, \hat{l})\) satisfies \((IC)\), since in the previous proof it was shown that, for \( l < l_0 \), \( \Pi'(l) \geq 0 \), so that it is not optimal to reduce \( l \) below \( \hat{l} \), and it is not possible to increase \( l \) above \( l_0 = \hat{l} \). Therefore, \((l_0^*, l^*) = (\hat{l}, \hat{l})\).

Proposition 6. For \( l_T \) defined in Proposition 4, we can rearrange problem \((P_R)\) as a univariate maximization problem:
\[
\begin{align*}
\text{Max}_{l_0} & \quad R_0(l_0) - w_0(l_0)l_0 + \delta[R(l(l_0)) - w(l_0)l(l_0)] \\
\text{s.t.} & \quad w_0(l_0) = \max \left\{ \frac{l_0(l(l_0))}{l(l_0)}(w_u(l(l_0)) - \overline{w}) + \overline{w} - \frac{C(l(l_0))}{l(l_0)}, \overline{w} \right\},
\end{align*}
\]
where the second restriction makes use of the fact that, by monotonicity, \( l = l_0 \), unless \( l_0 > l_T \).

This problem is identical to the problem where \( w_0 \) is unrestricted except for the fact that \( w_0 \geq \overline{w} \) is now required. Let \( w_0^T(l_0) = \overline{w} - \delta l(l_0)/l_0(w_u(l_0) - \overline{w}) \) be the unrestricted \( w_0 \).

If the restriction on \( w_0 \) is not effective at \((l_0^*, l^*)\), then \((l_0^*, l^*) = (l_0^*, l^*)\), since, if the restriction is effective for some other \( l_0 \), it only makes the latter less preferable by increasing \( w_0(l_0) \).

Suppose now that \( w_{0T}(l_0^*) < \overline{w} \), so that the restriction is binding at \((l_0^*, l^*)\):

1. For \( l_0 \) such that \( \frac{\delta(l(l_0))/l_0}{l(l_0)}(w_u(l(l_0)) - \overline{w}) + \overline{w} - \frac{C(l(l_0))}{l(l_0)} > \overline{w} \):
   
   1.a. If \( l_0 < l_u^* \), then \( l(l_0) = l_u(l_0) = l_0 \) and \( w_{0T}(l_0) = \overline{w} - \delta w_u(l_0) - C(l_0)/l_0 \), so that 
   \[
   w_{0T}'(l_0) = -\delta w_u(l_0) + \delta C(l_0)/l_0 - C(l_0) \geq 0,
   \]
   because of convexity of \( C \), and monotonicity.

   1.b. If \( l_0 \in (l_u^*, l_T) \), then \( l_u(l_0) < l(l_0) = l_0 \) and \( w_{0T}(l_0) = \overline{w} - \delta l_u(l_0) - C(l_0) \), so that 
   \[
   w_{0T}'(l_0) = \delta l_u(l_0) - C(l_0) + C'(l_0)l_0 \geq 0,
   \]
   because of convexity of \( C \).
2. For $l_0$ such that \( \frac{l_0(l(l_0))}{l(l_0)} (w_u(l(l_0)) - \overline{w}) + \overline{w} - \frac{C(l(l_0))}{l(l_0)} < \overline{w} \), \( w'_{0T}(l_0) = 0 \).

3. For \( \frac{l_0(l(l_0))}{l(l_0)} (w_u(l(l_0)) - \overline{w}) + \overline{w} - \frac{C(l(l_0))}{l(l_0)} = \overline{w} \), \( w'_{0T-}(l_0) = 0 \), and \( w'_{0T+}(l_0) \geq 0 \).

Therefore, since \( l_0^* < l_T \), \( w_{0T}(l_0) \) is increasing in \( l_0 \) for \( l_0 < l_0^* \). This means that if at the unrestricted optimum \( w_{0T}(l_0^*) \leq \overline{w} \), then \( w_{0T}(l_0) \leq \overline{w} \) for any \( l_0 < l_0^* \). Therefore, if \( l_0^* \) was preferred to an \( l_0 < l_0^* \) when \( w_0(l_0) < w_0(l_0^*) \), it will still be preferred when \( w_0(l_0) = w_0(l_0^*) = \overline{w} \).

It follows that, at the optimum, \( l_0 \) has to be at least as large as the unrestricted optimum. ■
References


