Working Paper 04-39
Economics Series 14
July 2004

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SHOULD NETWORK OPERATORS BE ALLOWED TO BUILD JOINT FACILITIES?[*]

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Abstract

In this paper, we address the issue of how the possibility for areements among network operators about building joint facilities affects their networks qualities, their profits and social welfare. We show that allowing the network operators to build joint facilities can make the network operators to increase their network qualities when they decide so simultaneously. When we analyze entry, only the incumbent increases his network quality. The main result is that network operators and the regulator coincide in thier decisions about how much the network operators should build jointly when the network operators decide simultaneously their network qualities. The same result arises when we analyze entry and the network operators are sufficiently differentiated. But, if there is entry and the network operators are not sufficiently differentiated, a regulator is hended to force the network operators to build joint facilities, what is very surprising from the current National Regulatory point of view.

[*] I gratefully acknowledge the research supervision of M. Angeles de Frutos, Pedro L. Marín and Tommaso Valletti. I would like to thank The Business School of the Imperial College of London for their hospitality while part of this research was conducted. The comments from Noemí Navarro, Ramon Xifré and Antonio G. Zaballos are very much appreciated. I also thank the Spanish Ministry of Education for financial support grant AP2000-0847. The usual disclimer applies

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1 Introduction

In this paper, we address the issue of how the possibility for agreements among network operators about building joint facilities affects their networks qualities, their profits and social welfare.

The following example picks up a situation where these kind of agreements arise. There are two network operators that enjoy full coverage in a country. At the same time, they have to decide their network qualities. The latters can be measured by the average intensity of the signal that consumers have available within the country. The consumers travel through the country with high frequency. The average intensity of the signal that consumers perceive is proportional to the amount of facilities that the network operators build. Up to now, the network operators can build facilities only on their own. So, each network operator has to decide non-cooperatively the amount of facilities they build. But, it is less costly if the network operators can build some of the facilities jointly. This is because they are avoiding the duplicity of some costs, e.g. it is more expensive to design two networks instead of one, and because if we take a given area of the country, it is cheaper to generate a given intensity with just one big network that to do so with two. Using the Erlang Formula we can easily see that the cost of the average intensity is decreasing with respect to the size of their facilities.[1] Outside the telecommunication industry, we can find more examples.[2]

About the issue we address, we find two opposite points of view. On one hand, the network operators argue that *"consumers will benefit through the delivery of faster, more innovative services at lower prices"*.[3] On the

[1]The Erlang Formula gives the size of a network such that it is always able to give service to the consumers of a telecommunication network operator.

[2]One possible example could be two academic departments that want to hold seminars independently. They want to invite the same professor for one of their seminars. This professor lives in USA and the academic departments are in Europe. If each academic department invites the professor on their own, each academic department has to pay a return flight ticket from USA to Europe. But, if both academic departments invite him to give a seminar one day in one of the academic departments and the day after in the other department, the academic departments have to pay only a return flight ticket from USA to Europe and a flight ticket within Europe instead of two return tickets from USA to Europe. Another example is found in the airline industry. In this industry, some airlines running the same route, e.g. British Airways and Iberia in the route Madrid-London, share planes. It is cheaper for them to fly a bigger airplane for both airlines passengers than to fly each airline its own smaller airplane.

[3]OFTEL (2002): "O2/T-Mobile 3G infrastructure sharing agreement: case

other hand, among National Regulatory Boards, we have found a complete different point of view, for example, OFTEL has expressed his view in these terms, *"the agreement may facilitate tacit collusion between the two parties and there may be spill-over effects which weaken competition at the retail level"*.[4]

So far, the effects of these agreements about building facilities are not clear on network qualities and prices. Depending on who is asked, we obtain very different answers. The academic literature on telecommunications neither helps very much. This literature has extensively studied topics like the Universal Service Obligation with important contributions in Valletti, Hoerning and Barros (2002), Anton, Vander Wide and Vettas (2002) and Chone, Flochel and Perrot (2002), or like the role of the interconnection charges on network competition, examples can be Laffont, Tirole and Rey (1998) and Armstrong (1998), but only recently it has started dealing with the topic of investment. So far, the most relevant article about investment on telecommunications is the paper by Valletti and Cambini (2003). They study the influence of the interconnection charges on network operators' investments. They challenge the profit-neutrality result that comes from the article by Laffont et al. (1998).[5] They show that the incentives to invest are influenced by the way the interconnection charges are set. When the quality of a network has an impact on the quantity of calls initiated by their own consumers, they obtain a result of tacit collusion in a model of two-part tariffs in the retail market.

Apart from the literature on telecommunications, we find similarities between our topic and the literature on cooperative R&D. Examples of this literature are d'Aspremont and Jacquemin (1988) and Katz (1986). In these papers, firms decide whether to carry on the research tasks separately or cooperatively through a joint venture. The main difference between the scenario we study and the cooperative R&D is that ours is more general. We allow the network operators to build facilities jointly and separately at the same time. Moreover, we also allow them to decide how much of these facil-

CMP/C1/N.38.370".

[4]OFTEL (2002): "O2/T-Mobile 3G infrastructure sharing agreement: case CMP/C1/N.38.370".

[5]In Laffont et al. (1998), network operators profits do not depend on the interconnection charges they pay each other if the network operators compete in the retail market with two-part tariffs.

ities is built separately and how much is built cooperatively.[6]

To study our problem, we use a similar model to the one in the article by Valletti and Cambini (2003), with the important differences that, in our model, the interconnection charges are fixed and the network operators can build jointly facilities.[7] We study three cases. First, the network operators are symmetric and they decide their network qualities simultaneously. Second, similar to the previous one but assuming network operators to be asymmetric. Their asymmetry is on demand, we consider that consumers have a higher perceived quality for one of the network operators. Finally, we allow for entry.

We show that the two opposite points of view about the impact of allowing the network operators to build facilities jointly may be motivated. Results depend on the timing of the decisions that the network operators make. If network operators decide first their qualities and second the amount of facilities they build jointly, allowing the network operators to build facilities jointly makes the network operators increases their network qualities. This is because it becomes "cheaper" for the network operators to invest on network quality. The opposite happens when the network operators decide firstly about the amount of facilities they build jointly and secondly about their network qualities. When the network operators are symmetric, they can use the cooperative agreement to neutralize the quality competition. For both network operators it is not profitable to deviate from the amount they decide to build jointly. Therefore, both network operators agree on choosing the minimum network quality. If the networks are asymmetric, the "collusive" agreement is less strong. The best considered network wants the other network to be as weak as possible while the least preferred network wants to be a stronger competitor. Both network operators end up investing less than with the other timing.

When we analyze entry, we show that the incumbent increases his quality but the entrant does not. The incumbent benefits because it is "cheaper" for her to invest, but the entrant always prefers to keep investing the amount that gives her the minimum network quality.

With respect to the impact on final prices, we find that if the network

[6] According to Katz (1986): *"It is important to recognize that in a more general setting allowing independent R&D may change the results dramatically."*

[7] The model that appears in Valletti and Cambini (2003) is the same model as in Laffont et al. (1998) with the difference that they introduce an investment stage prior to price competition.

operators are symmetric, prices of both network operators are the same as when network operators can only invest on their own. This is so because, although each network operator chooses different amounts than when they do not build jointly facilities, they always match each other in the amount they invest, getting the same network quality. This does not happen when we analyze the asymmetric and the entry cases. In the asymmetric case, we have got two possibilities. If network operators decide first the amount of facilities they build jointly and second their network qualities, when the best considered network has sufficient bargaining power the best considered network enjoys a higher price and the worst considered network a lower price. When we consider entry, the incumbent enjoys a higher price and the entrant a lower price.

But the most important finding is that the network operators and the regulator coincide on their decisions about the amount that network operators should build jointly when the network operators decide simultaneously their network qualities. This holds for the symmetric and for the asymmetric case. This result is restricted to the case when network operators decide first about their network qualities and second about the amount of facilities they build jointly. In this case, allowing them to build facilities jointly is desirable from the social point of view. Moreover, we would not need a regulator to keep guard on the process, because network operators decisions would be also right for social welfare. The opposite happens when the order of the network operators decisions is reversed. In such a context, allowing the network operators to build facilities jointly helps them to "collude" because they build networks of much lower quality.

Finally, when we analyze entry, if network operators are sufficiently different in the product they offer, their decisions and the regulator decision about how much the network operators should build jointly coincide. But, if network operators are not sufficiently different, in the product they offer, they decide a lower quantity compared to the social optimum. A regulator could be needed in order to force the network operators to build joint facilities. As we mentioned above, this is an unexpected result from the National Regulatory Boards point of view.

This paper is organized as follows. In Section 2, we describe the basic model and we solve for the last stage of the game (price competition). In section 3, we solve the case with symmetric network operators. In section 4, we solve the case with asymmetric network operators. In section 5, we analyze entry. In section 6, we conclude.

2 The Basic Model

2.1 Demand and Cost Structure

We use a model similar to the one used in Valletti and Cambini (2003) with the differences that in our model interconnection charges are fixed, consumers demands are inelastic and network operators can build joint facilities.

There are two network operators, A and B, located at the ends, $x_A = 0$ and $x_B = 1$ of a segment $[0, 1]$ that represents a country. Each network operator competes for the calls of the individuals.

The network operators play a three stages game. At stage one, the network operators decide simultaneously and noncooperatively the infrastructure levels of their networks. We denote the network operator A infrastructure level by I_A and the network operator B infrastructure level by I_B. The infrastructure levels measure the quality of the networks.[8] We suppose that by regulation there is a minimum infrastructure level denoted by $\underline{I}$.

The network operators can build jointly some of the facilities. We assume that there are economies of scale when they build jointly facilities. We call I_s the infrastructure level that each network operator enjoys from the facilities that they build jointly. This amount can never exceed any of the infrastructure level decided by the network operators, $I_s \leq \min\{I_A, I_B\}$.[9]

At stage two, the amount of I_s is decided either cooperatively by the network operators or by a regulator. At stage three, given the network configuration that arises from previous stages, the network operators simultaneously and independently choose prices p_i, $i = A, B$, and the network operators get the final payoffs.[10]

We assume, for simplicity, that the network operators marginal costs are 0. For the infrastructure level costs, we assume the function $F_i(I_i, I_s) = (I_i - \gamma I_s)^\alpha$, $\alpha > 1$, $\gamma < 1$, $i = 1, 2$.[11] Both functions are increasing and

[8]Following the example of the introduction I_A and I_B can be interpreted as the average intensity of the signal available to consumers from each network operator within the country

[9]$I_s \leq \min\{I_A, I_B\}$ reflects the fact that any network operator cannot be forced to build more facilities jointly that the necessary amount to get the network infrastructure level that the network operator wants.

[10]To study our problem deeply, we use other games with the same stages but with different timings.

[11]We construct the cost functions for infrastructure level as follows: We consider I_i as the total network operator i infrastructure level. Then, $I_i = k_i + I_s$, where k_i is the

convex with respect to I_i, decreasing with respect to I_s and with negative cross derivative with respect to I_i and I_s.

The consumers are a unit mass uniformly located along the segment $[0, 1]$. We assume that each consumer has a unit demand for calls. When a consumer located at x in the segment joins a network operator located at x_i, he enjoys a utility given by:

$$u_i(I_i) + v_0 - \frac{1}{\sigma} \mid x - x_i \mid - p_i;\ i = A, B,\ x \in [0, 1]$$

where v_0 is a fixed surplus from subscribing to any network operator. We assume that v_0 is large enough to all consumers connected to a network operator. The utility that network operators infrastructure level gives to consumers is $u(I_i)$, (hereafter u_i).[12] Any increase in the level of a network operator infrastructure level gives more utility to his consumers, however the raise is lower as the level of the network operator infrastructure level is higher. To be consistent with these assumptions, we assume that u_i is increasing and concave with respect to I_i. Finally σ measures the degree of substitution among the network operators, i.e. the intensity of price competition.[13]

We look for the Pure Strategy Subgame Perfect Nash Equilibria of the game.

2.2 Market Shares

From above, a consumer located at x subscribing to network operator A enjoys utility:

$$u_A + v_0 - \frac{1}{\sigma} x - p_A$$

network operator i infrastructure level build on his own and I_s and the part of investment build jointly. The cost functions for infrastructure level is $F_i(k_i, I_s) = (k_i + \delta I_s)^\alpha$, $\delta < 1$. Notice that when they build jointly is cheaper because of the economies of scale. From equation $I_i = k_i + I_s$, we know that $k_i = I_i - I_s$. If we replace the expression in the costs functions we obtain a new cost function $F_i(I_i, I_s) = (I_i - (1 - \delta)I_s)^\alpha$. If we call $\gamma = 1 - \delta$, we obtain our cost functions $F_i(I_i, I_s) = (I_i - \gamma I_s)^\alpha$.

[12]Notice that the network operators infrastructure level utility, u_i, is only a function of I_i, I_s does not appear. This means that we assume that the consumers only care about the level of infrastructure they enjoy and they do not care at all about how the infrastructures are built.

[13]σ is related to the inverse of transport cost, i.e., the notional costs that consumers pay when they purchase a variety distant from their ideal.

and if he subscribes to network operator B, he enjoys utility:

$$u_B + v_0 - \frac{1}{\sigma}(1 - x) - p_B$$

The indifferent consumer is one located at θ such that:

$$u_A + v_0 - \frac{1}{\sigma}\theta - p_A = u_B + v_0 - \frac{1}{\sigma}(1 - \theta) - p_B$$

Thus:

$$\theta = \frac{1}{2} + \sigma(\frac{(u_A - u_B) + (p_B - p_A)}{2})$$

Given that the indifferent consumer is located at θ in the segment and the localization of the network operators ($x_A = 0$ and $x_B = 1$), the network operator A market share is $\theta_A = \theta$ and the network operator B market share is $\theta_B = 1 - \theta$.

2.3 Price Competition

In this last stage, the infrastructure levels are fixed, hence, the network operator i has to solve the following maximization problem:

$$\max_{p_i} \Pi_i = p_i(\frac{1}{2}+\sigma(\frac{(u_i - u_j) + (p_j - p_i)}{2}))-(I_i-\gamma I_s)^{\alpha},\ for\ i = A, B\ and\ j \neq i$$

Taking the derivative with respect to p_i, we obtain:

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{1}{2} + \sigma(\frac{(u_i - u_j) + (p_j - 2p_i)}{2}) = 0,\ i = A, B$$

From the system of equations, equilibrium prices are:

$$p_i^* = \frac{1}{\sigma} + \frac{(u_i - u_j)}{3}\ i = A, B$$

The equilibrium prices reflect the two sources of differentiation in the industry. First, as the degree of substitution, σ, is lower, both network operators can charge higher prices. Second, if the network operators enjoy a different amount of infrastructure level, the network operator with the higher infrastructure level can charge higher price than his rival.

3 Infrastructure level: Symmetric Network Operators

3.1 Social Planner Benchmark

A useful benchmark is given by the infrastructure levels, I_i, and by the infrastructure level from the facilities that they build jointly, I_s, that would be chosen by a benevolent social planner. A benevolent social planner solves the following maximization problem:

$$\max_{I_i,s} W = \frac{u_i + v_0}{2} - (I_i - \gamma I_s)^\alpha$$

$$s.t.\ I_s \leq \min\{I_i, I_j\}$$

As both network operators are symmetric, each network operator serves half of consumers, furthermore, any consumer enjoys a utility $u_i + v_0$ from consuming from any of both network operators when their infrastructure level is I_i. Then, the total consumers utility per network operator is $(u_i + v_0)/2$. Both network operators have to pay a cost $(I_i - \gamma I_s)^\alpha$ to be able to offer a infrastructure level I_i to their consumers when a part, I_s, of their infrastructure level comes from the facilities that they have built jointly.

It can be checked out that the derivative of W with respect to I_s is positive. This means that the social planner wants I_s to be as high as possible, i.e., $I_s = \min\{I_i, I_j\}$. Once this is clear, we obtain the following result:

Proposition 1 *If the network operators can build facilities jointly, the social planner optimum is $I_s^P = I_P$ and $I_i^P = I_j^P = I_P$, where I_p solves:*

$$\frac{1}{2}\frac{\partial u_i}{\partial I_i} - \alpha(1-\gamma)^\alpha I_i^{\alpha-1} = 0$$

In equilibrium $I_s^P = I_i^P = I_j^P = I_p$. We could think that $\min\{I_i, I_j\}$ could be I_j, and $I_i^P > I_j^P$. But if we think that the network operators choose the same amount of infrastructure level when they cannot build jointly facilities and that the social planner would ask the network operator with $\min\{I_i, I_j\}$ for a higher infrastructure level to relax the constraint $I_s \leq min\{I_i, I_j\}$ to allow the network operators to build jointly more facilities. Then, we would find out a situation where $I_j^P > I_i^P$. The social planner asks both network operators to have the same infrastructure level. Moreover, the social planner wants both network operators to build all their facilities jointly.

With the introduction of the possibility for the network operators of building jointly facilities, we have found a cheaper way of getting infrastructure level. Therefore, the social planner just chooses the most efficient way of building facilities. This brings us to the following result:

Proposition 2 *Building facilities jointly is welfare enhancing.*

3.2 Infrastructure level competition: The amount of facilities network operators can build jointly is set by a regulator

In this subsection, given the equilibrium prices of the last stage of the game and the decision of the regulator, both network operators decide their infrastructure levels. As a benchmark, first, we check the infrastructure level when the network operators are not allowed to build jointly facilities, $I_s = 0$. In this case, each network operator has to maximize profits, Π_i^*:

$$\max_{I_i} \Pi_i^* = \frac{1}{2\sigma} + \frac{(u_i - u_j)}{3} + \sigma(\frac{(u_i - u_j)^2}{18}) - I_i^\alpha$$

Applying symmetry to the first order condition, the equilibrium investment satisfies the following condition:

$$I_i^* \ is \ I_i \in \Re_+ \ / \ \frac{1}{3}\frac{\partial u_i}{\partial I_i} - \alpha I_i^{\alpha-1} = 0$$

The infrastructure level that the network operators choose in equilibrium is far lower from the social optimum infrastructure level.[14] With this in mind, we compare it with the possibility of building jointly facilities and how it does affect the infrastructure level. First, recall that the constraint $I_s \leq \min\{I_A, I_B\}$ becomes in the symmetric case $I_s \leq I_i$. Depending on the quantity, I_s, set by the regulator, we distinguish two cases: $I_s < I_i$ and $I_s = I_i$. In the first case, the regulator sets a quantity lower than

[14]The social planner optimum infrastructure level is $\frac{1}{2}\frac{\partial u_i}{\partial I_i} - \alpha I_i^{\alpha-1} = 0$ higher than the privately chosen investment $\frac{1}{3}\frac{\partial u_i}{\partial I_i} - \alpha I_i^{\alpha-1} = 0$.

the infrastructure level chosen by the network operators, in the second case the regulator sets a quantity equal to the infrastructure level chosen by the network operators. In any case we can conclude:

Proposition 3 *If the network operators can build joint facilities and a regulator sets the quantity, I_s, the network operators choose higher infrastructure levels than when they cannot build facilities jointly.*

The intuition about this result is simple. When the network operators can build facilities jointly, the marginal cost of building those facilities is lower due to the economies of scale they get. At the same time, consumers are not affected by the way the facilities are built but by the quality of the networks measured by the amount of infrastructure level. Thus, the marginal revenue does not change while the marginal cost is lower, therefore the network operators choose higher infrastructure levels for their networks.

We think that allowing the network operators to build jointly facilities is good from a social point of view, because it yields higher infrastructure levels. But, a question that we have not answered yet is the quantity that the regulator should set to reach the closest infrastructure level to the social planner optimum level. This quantity is $I_s = \min\{I_i, I_j\}$, as we claim in the following result:

Proposition 4 *The regulator optimum choice is $I_s = \min\{I_i, I_j\}$. The network operators always choose a lower infrastructure levels compared with the social planner optimum for any I_s.*

Because the network operators marginal cost of infrastructure levels decrease with the possibility of building facilities jointly, the network operators end up choosing higher infrastructure levels. This is, in general, very convenient, because we can improve the social welfare. As the social welfare increases as I_s is higher, then, the regulator should choose $I_s = \min\{I_i, I_j\}$ to get the highest possible social welfare.

In this case, improving the social welfare does not mean that we are closer to the social planner optimum. This is so because the possibility for the network operators of building jointly infrastructures also makes higher the social planner optimum infrastructure level. The social planner internalizes the lower marginal cost of the infrastructure level in the same way as the network operators do.[15] The marginal revenue functions remain constant for the social planner and for both network operators. If we pay attention, we can see that the marginal revenue is always greater for the social planner in constant proportion. This means that the proportions between the network operators optimum infrastructure level and the social planner optimum remains constant, which means that the difference between both optima are greater.

We also should be concerned about how building jointly facilities affects the network operators profits. At first sight, the answer is not very clear. On one hand, it is "cheaper" for the network operators to get infrastructure levels due to the economies of scale from building jointly facilities. On the other hand, as the marginal cost of infrastructure level is lower, they also choose higher infrastructure levels. We have two opposite effects and depending on the winning effect, the network operators make higher or lower profit. In the next proposition we have the solution to this trade-off:

Proposition 5 *If network operators are allowed to build facilities jointly and the regulator sets the quantity they can build jointly, I_s, the network operators enjoy higher profits.*

Finally, the network operators make more profit when they can build facilities jointly. Although, in equilibrium, they choose higher infrastructure levels because the marginal cost of investing is lower, the savings due to the economies of scale are higher. The reason why this occurs lies in the concavity of the consumers utility function. Given that marginal utility is decreasing, it is not worthy for the network operators to expand their infrastructure levels very much.

From the analysis made up to now, we can conclude that allowing the network operators to build jointly facilities when the quantity to be built jointly is set by a regulator is welfare enhancing and, furthermore, makes the network operators profits higher.

[15]If we check out the social planner problem and the network operators problem, we see that they have got the same infrastructure level cost functions $(I_i - \gamma s)^\alpha$ and therefore the same infrastructure level marginal cost function

3.3 Infrastructure level competition: The network operators decide the amount of facilities to be built jointly

We analyze the situation where network operators can build jointly facilities and they also decide cooperatively about so.[16] In order to do so, we propose the model of the previous subsection with the difference that the amount of facilities that the network operators build jointly is decided cooperatively by bargaining among themselves.

We use a game where at the first stage the network operators choose non-cooperatively their networks infrastructure levels, I_i. At the second stage they decide through bargaining the amount of facilities they build jointly. As before, we represent it by I_s, the infrastructure level that each network operator enjoys from the infrastructure they build jointly. At the last stage, the network operators decide non-cooperatively their prices and final payoffs are realized.

For the cooperative stage, the network operators play a Bargaining Game, where the network operators build nothing jointly as the disagreement point, $I_s = 0$.

The last stage, where the network operators decide their prices, is identical to the case where a regulator decides the amount of facilities that the network operators can build jointly.

Following backward induction procedure, we continue solving the bargaining stage by means of the Nash Bargaining Solution. We solve the following problem:

$$\max_{I_s} \ \Pi_A^{\beta} \Pi_B^{1-\beta}$$

$$s.t. \ I_s \leq \min\{I_A, I_B\}, \ \beta \in (0,1)$$

where

$$\Pi_A = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma \frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^{\alpha}$$

[16] As we have shown in the introduction, National Regulatory Boards as OFTEL, warn about possible damages in competition when network operators build jointly facilities and they decide cooperatively about so.

and

$$\Pi_B = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^\alpha$$

From the maximization problem above, we can conclude:

Lemma 1 *The Nash Bargaining Solution at the Bargaining Stage is* $I_s = min\{I_A, I_B\}$ *or* $I_s = 0$.

The network operators find that any increase in I_s reduces the marginal cost of the infrastructure levels. Then, it seems that the solution is $I_s = \min\{I_A, I_B\}$ conditional on the fact that the disagreement status quo is not preferred by any of the network operators. Otherwise, the solution would be $I_s = 0$.

Once we have got the possible solutions to the Bargaining Problem, we can solve the last stage of the game. In this stage, the network operators decide non-cooperatively their infrastructure levels. Both firms have to solve the following maximization problems:

$$\max_{I_i} \Pi_i^* = \frac{1}{2\sigma} + \frac{(u_i - u_j)}{3} + \sigma\frac{(u_i - u_j)^2}{18} - (I_i - \gamma I_s)^\alpha, \; i = A, B, \; j \neq i$$

$$s.t. \; I_s = \min\{I_A, I_B\} \; or \; I_s = 0$$

The reader should note that this problem is identical to the problem we have solved when we have analyzed the case where a regulator sets the quantity they could build jointly. The only difference now is that I_s can be 0 or $\min\{I_A, I_B\}$.

Proposition 6 *In a SPNE, network operators choose an infrastructure level identical to the case where a regulator sets the amount of facilities that they can build jointly.*

We should point out that a key feature for this result is that the network operators make more profits when $I_s = \min\{I_A, I_B\}$ than when $I_s = 0$. This is so, because although they choose a higher infrastructure level, the cost savings due to amount of facilities that they build jointly is far higher than the costs due to the higher infrastructure level. Once this point is clear, the intuition about the result is rather the same than in the case where a regulator sets the amount of facilities that the network operator can build jointly.

This result shows a very important feature in this context, it is the same that a regulator chooses the amount of facilities that the network operators can build jointly, as this amount is chosen by the network operators cooperatively. Then, we can conclude that a regulator is not required to keep guard the network operators election of I_s. This is a very good result, because we are avoiding the possible distortions of the regulation.[17]

We move to check the robustness of the last result to games with a different timing. We consider two new types of game. In the first type, the network operators, at the first stage, decide cooperatively the amount they build jointly. At the second stage, the network operators decide their infrastructure levels. Finally, they decide non-cooperatively their final prices.

In the second type of game, at first stage, simultaneously, they decide cooperatively the amount they build jointly and non-cooperatively their infrastructure levels. Finally, again, they decide non-cooperatively their final prices.

If we solve these games and we compare them among themselves and with the previous game, we see that:

Proposition 7 *Let network operators choose simultaneously their infrastructure levels and the amount of facilities they build jointly. Then, the results are identical to the previous scenario analyzed. On the other hand, let network operators decide first the amount of facilities they build jointly and afterwards their infrastructure levels. Then, they underinvest by choosing the minimum infrastructure levels,* $I_i^* = I_j^* = \underline{I}$.

This result shows that the timing of the decisions matters. Depending on the timing, we have two complete different results. On one hand, we have got that the network operators choose the right decisions from a social point of view, but on the other hand, if the network operators choose first the amount of facilities they build jointly, they have an incentive to "collude" setting networks with the minimum quality requirements $\underline{I}$. This "collusion" agreement comes from the fact that the network operators know from the

[17]These distortions mostly come from the regulator lack of information about the network operators. See Laffont and Tirole *"A theory of incentives in procurement and Regulation"* for a deep discussion.

bargaining process that they will build all the facilities jointly and they will not build anything on their own. Given this, the best they can do is to build facilities with the minimum quality.

From the results obtained up to now, we can conclude that the National Regulatory Boards should allow the network operators to build facilities jointly. But, they should control the timing of the network operators decisions, giving no room to the possible "collusion".[18]

4 Infrastructure levels: Asymmetric Network Operators

4.1 Infrastructure level competition: the amount of facilities that the network operators can build jointly is set by a regulator

We consider the same model than before but with an important difference. We propose an asymmetry that comes from the demand side. We assume that one of the network operators, for example the network operator A, gives more utility to consumers when both network operators have chosen the same infrastructure level.[19] In our model, this assumption is represented by $u_A(\tau_A, I_A) = \tau_A u(I_A)$ and $u_B(\tau_B, I_B) = \tau_B u(I_B)$ (hereafter $u_i(\tau_i, I_i) = u_i$), where $\tau_A > \tau_B > 0$ and $u(I_i) = 0$ if $I_i = 0,\ i = A, B$.[20]

First we check the infrastructure levels when the network operators are not allowed to build jointly facilities. Given equilibrium prices of the last

[18]There is another option for the National Regulatory Boards. This is that the National Regulator Boards directly set I_s, the amount of facilities that they can build jointly. But, this option can be seen as too intrusive. Again problems of information. See footnote 17.

[19]Our new model can help us to understand investment behavior in the past, when the first duopolies were set up in the mobile telephony. In these cases, one of the network operators almost always was owned by the incumbent of the fixed telephony while the other network operator came from firms not very well known by consumers. This better knowledge by consumers from the network operators owned by the incumbents of the fixed telephony was and is still and advantage for these network operators because consumers have a perceived quality for them.

[20]A similar setting is used in the paper by Carter and Wright (1999) where they study the influence of the brand loyalty on the competition among network operators. The most important difference with this paper is that the network operators do not take decisions about the investments on their networks.

stage of the game, the network operator A has to maximize profits, Π_A^*:

$$\max_{I_A} \ \Pi_A^* = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma(\frac{(u_A - u_B)^2}{18}) - I_A^\alpha$$

The network operator B also maximizes profits, Π_B^*:

$$\max_{I_B} \ \Pi_B^* = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma(\frac{(u_A - u_B)^2}{18}) - I_B^\alpha$$

As $u_A > u_B \ \forall I_A = I_B$, in equilibrium, both network operators cannot enjoy the same infrastructure level.[21] The remaining question is who chooses a higher infrastructure level. We give the solution in the following lemma:

Lemma 2 *Let network operators be not allowed to build joint facilities. In equilibrium, $I_A^* > I_B^*$.*

The network operator A chooses a higher infrastructure level than the network operator B for three reasons. Firstly, the network operator A enjoys more direct marginal revenue from its infrastructure level, measured by $\frac{1}{3}\frac{\partial u_A}{\partial I_A}$, than the network operator B, $\frac{1}{3}\frac{\partial u_B}{\partial I_B}$. [22] Secondly, the marginal cost functions are equal for both network operators. And lastly, the network operators have incentives to be differentiated in the levels of investment they choose, this effect is measured by the indirect marginal revenues:

$$\frac{\sigma}{9}(u_i - u_j)\frac{\partial u_i}{\partial I_i}$$

These indirect marginal revenues induce to choose a higher infrastructure level to the network operator A and lower to the network operator B. This last effect is milder when the degree of differentiation, σ, is lower.

Once we have seen what happens with the network operator infrastructure levels when they are not allowed to build jointly facilities, we can move to check how the network operator infrastructure levels are affected by the possibility for the network operators of building jointly facilities.

[21] If we check the first order conditions, we can conclude that both equations cannot be 0 at the same time when $I_A = I_B$.

[22] $u_A > u_B, \forall I_A = I_B$ and $u_i = 0$ if $I_i = 0 \Rightarrow \frac{\partial u_A}{\partial I_A} > \frac{\partial u_B}{\partial u_B} \ \forall I_A = I_B$.

Before starting the analysis, note that the previous restriction $I_s \leq \min\{I_A, I_B\}$ is present in this case too.

As in the symmetric case, the regulator sets I_s to the maximum possible amount $I_s = \min\{I_A, I_B\}$, because for the regulator I_s is just a cost saving factor.[23] The network operators solve the following maximization problems:

$$\max_{I_A} \ \Pi_A^* = \frac{1}{2\sigma} + \frac{u_A - u_B}{3} + \sigma(\frac{(u_A - u_B)^2}{18}) - (I_A - \gamma I_s)^\alpha$$

$$s.t. \ I_s \leq \min\{I_A, I_B\},$$

for the network operator A, and for the network operator B:

$$\max_{I_B} \ \Pi_B^* = \frac{1}{2\sigma} + \frac{u_B - u_A}{3} + \sigma(\frac{(u_A - u_B)^2}{18}) - (I_B - \gamma I_s)^\alpha$$

$$s.t. \ I_s \leq \min\{I_A, I_B\}$$

Notice that in equilibrium $\min\{I_A, I_B\} = I_B$. As both network operators have the same cost function and network operator A is still preferred by consumers, I_B cannot be higher than I_A in equilibrium. Given this fact and the maximization problems, we can conclude:

Proposition 8 *When network operators can build jointly facilities and a regulator sets the quantity that they can build jointly, the network operators choose higher infrastructure levels for their networks.*

The reasons for the raising of both network operators infrastructure levels are the same as in the symmetric case. If the network operators can build jointly facilities, they get lower marginal costs for the infrastructure levels while the marginal revenue remains constant. That means that both network

[23]In the planner problem, I_s only appears in the cost function that is represented by $MS_A(I_A - \gamma I_s)^\alpha + MS_B(I_B - \gamma I_s)^\alpha$, where MS_A and MS_B are the network operator A and B market shares. Given this, it is straight forward to see that the social planner wants I_s to be as high as possible.

operators increase their infrastructure levels compared to the case where they can build facilities only on their own.

Again, allowing the network operators to build jointly facilities is welfare enhancing because the infrastructure levels increase.

Another important remaining issue is how the possibility of building jointly affects the existing gap between the two network operators equilibrium infrastructure levels, $I_A^* > I_B^*$. We answer this question in the following proposition:

Proposition 9 *Let be I_A^*, I_B^*, respectively, the network operator A and B equilibrium infrastructure levels when they do not build jointly facilities and let be I_A^{**}, I_B^{**} are the equilibrium infrastructure when they do, then:*

$$I_A^* - I_B^* > I_A^{**} - I_B^{**} \ \forall \alpha \geq 2$$

This means that when the regulator sets a quantity I_s, the network operator B infrastructure level, I_B, follows a process of catch-up of the network operator A infrastructure level, I_A. This is because the network operator B infrastructure level binds the amount of facilities that the network operators can build jointly. This gives to network operator B an extra incentive to choose a much higher infrastructure level. In terms of the impact of I_s on prices, as the infrastructure level gap is closer, the network operators products are more homogeneous and therefore the prices also get closer. The network operator A enjoys a lower price and viceversa for the network operator B.

Apart from what happens to prices, allowing operators to build jointly facility is consumer surplus enhancing for two reasons: both network operators consumers enjoy more infrastructure levels and because the utility gain from the network operator A price drop can never be overcome by the network operator B price increase.[24]

[24] The competition between the network operators make impossible for them to appropriate the new surplus via prices.

4.2 Infrastructure level competition: The network operators decide the amount of facilities to be built jointly

As we have made in the symmetric case, we consider the case where I_s is decided cooperatively by the network operators. Again, we are concerned about how the decision about I_s may damage the competition among the network operators.

We use the same model we have used in the symmetric case when the network operators also decide about the amount of facilities they build jointly, but we introduce the same asymmetry on demand as when a regulator decides about I_s. We assume that one of network operators, network operator A, gives more utility to consumers when both network operators have the same infrastructure level. Recall that, $u_A(\tau_A, I_A) = \tau_A u(I_A) = u_A$ and $u_B(\tau_B, I_B) = \tau_B u(I_B) = u_B$ where $\tau_A > \tau_B > 0$ and $u(I_i) = 0$ if $I_i = 0,\ i = A, B$.

Again at the last stage, where the network operators decide their prices, this case is identical to the one where there is a regulator deciding the amount of facilities that the network operators can build jointly.

Following backward induction procedure, we continue solving the second stage of the first game, assuming that they agree on the Nash Bargaining Solution. We compute it solving the following problem:

$$\max_{I_s} \Pi_A^{\beta} \Pi_B^{1-\beta}$$

$$s.t.\ I_s \leq \min\{I_A, I_B\},\ \beta \in (0,1),\ u_A > u_B\ \forall I_A = I_B \neq 0,$$

where

$$\Pi_A = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^{\alpha}$$

and

$$\Pi_B = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^{\alpha}$$

The solution is the same that in the symmetric case, either $I_s = \min\{I_A, I_B\}$ or $I_s = 0$. In principle, both network operators want the amount of facilities that they build jointly to be as high as possible. The reason is that any increase in I_s reduces the marginal costs of the infrastructure levels. It seems that the solution is $I_s = \min\{I_A, I_B\}$, but it could happen that the network operators choose the disagreement point, in that case, the solution would be $I_s = 0$.

With this result, we can solve the first stage of the game. In this first stage, the network operators decide non-cooperatively their infrastructure levels. They solve:

$$\max_{I_A} \Pi_A^* = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^\alpha$$

$$s.t.\ I_s = \min\{I_A, I_B\}\ or\ I_s = 0$$

$$u_A > u_B\ if\ I_A = I_B \neq 0$$

for the network operator A. For the network operator B:

$$\max_{I_B} \Pi_B^* = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^\alpha$$

$$s.t.\ I_s = \min\{I_A, I_B\}\ or\ I_s = 0$$

$$u_A > u_B\ if\ I_A = I_B \neq 0$$

If we compare these maximization problems with the maximization problems where there is a regulator, we see that they are basically identical with the difference that now the network operators can also choose $I_s = 0$, the disagreement point. But, no network operator is interested in choosing the disagreement point, because, although allowing the network operators to build jointly facilities increases the infrastructure levels (which may suppose a loss in profit via lower revenue for the network operator A) and more cost, nevertheless, the cost savings, due to the economies of scale, are much higher. The network operators end up with higher profits. Given this, we can conclude:

Proposition 10 *Let network operators decide first about their infrastructure levels and second they decide cooperatively the amount of facilities they build jointly. Then, they choose the same amount of infrastructure levels as when the amount of facilities they build jointly is decided by a regulator.*

The regulator and the network operators choose the same. This is important because if the network operators follow this timing on their decisions, we do not need to apply any regulation to this industry.

To check the robustness of this result with respect to the timing of the game, we solve the other two games we have proposed before, when we have analyzed the situation with symmetric network operators. If we compare the results obtained with the results obtained in the previous game, we can see that:

Proposition 11 *Let network operators choose simultaneously their decisions about their infrastructure levels and the amount of facilities they build jointly. Then, the results are identical to the previous scenario analyzed. Let network operators decide, first, about the amount of facilities they build jointly and afterwards decide their infrastructure levels. Then, in this case, both network operators choose a lower infrastructure level.*

Again, the timing of the decisions matters. Although the differences in the results about the infrastructure levels is not so large as it was in the symmetric case.[25]

In this case, we find that the best considered network operator forces the other network operator to choose a lower infrastructure level. The best considered network operator agrees in building jointly the maximum possible amount of infrastructures but in exchange for it, the worst considered network has to lower his infrastructure level. With this action, the best considered network operator gets a weaker competitor. With it, the best considered network operator does not need to invest in his network as much as he does in the other cases.

From the last result, we can conclude that the National Regulatory Boards should allow the network operators to build facilities jointly. But, they should control the timing of the network operators decisions to force them to choose the infrastructure levels as high as possible.[26]

[25] Recall that in the symmetric case, when the network operators chose first the amount of facilities they build jointly and second their infrastructure level, they chose the minimum infrastructure level, $\underline{I}$.

[26] Idem footnote 18.

5 Entry

5.1 Infrastructure level competition: The amount of facilities that the network operators can build jointly is set by a regulator

In order to analyze entry, we now study the symmetric model with the difference that the network operators take their infrastructure level decisions sequentially.[27] We assume that the network operator A is the incumbent and is, therefore, the first to choose infrastructure level and the network operator B is the entrant and the second to choose infrastructure level. Our aim is to know how both network operator invest under the possibility of building jointly facilities and compare it with the case where no facilities can be built jointly.[28] As in the previous subsections, we start looking at what happens when the network operators are not allowed to build jointly facilities. The entrant has to solve the following maximization problem:

$$\max_{I_B} \ \Pi_B^* = \frac{1}{2\sigma} + \frac{u_B - u_A}{3} + \sigma\frac{(u_A - u_B)^2}{18} - I_B^\alpha$$

If we take the derivative with respect to I_B we obtain the entrant reaction function:

$$\frac{\partial \Pi_B^*}{\partial I_B} = \frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha I_B^{\alpha-1} = 0$$

Following the backward induction procedure, we set up the incumbent maximization problem, given the network operator B reaction function:

$$\max_{I_A} \ \Pi_A^* = \frac{1}{2\sigma} + \frac{u_A - u_B}{3} + \sigma\frac{(u_A - u_B)^2}{18} - I_A^\alpha$$

$$s.t. \ \frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha I_B^{\alpha-1} = 0$$

From these maximization problems, we derive the following result:

[27] The paper by Henkel (2002) proves that this is a very good setting to analyze entry, because the infrastructure level are strategic substitute and therefore, there is not problems of commitment on the decision made by the incumbent.

[28] In this case, for entry, we describe a situation where the entrant has already taken his decision of entry but he has to take all the rest of decisions, in our case, decisions as his infrastructure level or his retail prices

Lemma 3 *When the network operators decide sequentially their infrastructure level, the incumbent chooses a higher infrastructure level than the entrant, $I_A^* > I_B^*$. Moreover, the entrant chooses the minimum infrastructure level $\underline{I}$ that a network operator has to supply.*

From this result, we can see that the incumbent profits from his first-mover advantage to get the best position in the market. When the entrant has to decide his infrastructure level, he sees that the only profitable possibility is to choose a lower infrastructure level than the incumbent, becoming a lower quality network operator and due to it a weak competitor.[29] It is important to stress that the gap in the infrastructure level is wider as the degree of substitution between networks is lower. As the degree of substitution decreases, it is more difficult to make profit throughout the infrastructure level differentiation. As the incumbent can choose first, he tries to maintain profits raising his infrastructure level to get a wider gap between his infrastructure level and the entrant infrastructure level.

The other important result is that the network operators choose the minimum infrastructure levels they can. The incumbent chooses first and he always wants to maintain a gap in the infrastructure level equal to:

$$u_A = u_B + \frac{3}{\sigma}(\frac{1}{3} - \frac{\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}})$$

This means that the incumbent wants an infrastructure level such that it gives the same utility to consumers plus the optimal gap in the infrastructure levels. This term is just a negative function of the entrant infrastructure level. As the difference between infrastructure levels damages the entrant profits, the entrant chooses the minimum amount of infrastructure level he can. If he chose a higher infrastructure level, given the incumbent reaction, he would get a lower profit, for the revenues would remain constant and the costs would raise. This is a very worried result for the social welfare, because these infrastructure levels are very far from the ideal. As a consequence, the infrastructure levels should be boosted using any remedy.[30]

[29] There is a possibility that the entrant chooses infrastructure level to jump ahead of the incumbent. That is $u_B > u_A$ $\forall I_A = I_B$ sufficiently large, but this possibility should be ruled out because it is unlikely.

[30] It is good to point out that the network operators need to be horizontally differentiated

Once we know how the network operators behave when they cannot build facilities jointly, we try to know what happens to network operators when we consider entry and the incumbent and the entrant can build facilities jointly. Again the maximum infrastructure level, I_s, that each network operator can enjoy from the facilities build jointly is set by a regulator. As in the other cases, the network operators can build jointly as much as the smallest infrastructure level chosen by them, $I_s \leq \min\{I_A, I_B\}$.

As in the other cases, the regulator will always set I_s to the maximum possible amount, because for the regulator, I_s, is just a cost saving factor.[31] Given this, the entrant has to solve the following maximization problem:

$$\max_{I_B} \ \Pi_B^* = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^\alpha$$

$$s.t.\ I_s = \min\{I_A, I_B\}$$

To solve this problem, first we assume that $\min\{I_A, I_B\} = I_B$, which is the result in equilibrium, if they play a Stackelberg game. That means that the entrant maximization problem becomes:

$$\max_{I_B} \ \Pi_B^* = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (1-\gamma)^\alpha I_B^\alpha$$

We take the derivative with respect to I_B:

$$\frac{\partial \Pi_B^*}{\partial I_B} = \frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha(1-\gamma)^\alpha I_B^{\alpha-1} = 0$$

With the entrant reaction curve, we can set up the incumbent maximization problem:

$$\max_{I_A} \ \Pi_A^* = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_B)^\alpha$$

$$s.t.\ \frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha(1-\gamma)^\alpha I_B^{\alpha-1} = 0$$

Given these maximization problems, we can get the following result:

because they get higher profits. An example where this has happened is the Spanish mobile telecommunication industry where the last entrant, Amena, has tried to attract different groups of consumers from his competitors groups of consumers.

[31] See the social planner problem in the section where the network operators are symmetric and choose simultaneously infrastructure levels.

Proposition 12 *Let network operator be allowed to build jointly facilities under entry, and let the quantity to be built jointly to be set by a regulator. Then the entrant chooses the minimum infrastructure level, $\underline{I}$, that a network operator has to supply and the incumbent expands his infrastructure level.*

This result tells us that allowing network operators to build jointly facilities is not a good instrument for the entrant to become a tougher competitor. The incumbent chooses a higher infrastructure level because the marginal costs of the infrastructure level is lower, while the entrant continues choosing the minimum infrastructure level he can. The positive aspect is that the incumbent raises his infrastructure level, which is welfare enhancing.

5.2 Infrastructure level competition: The network operators decide the amount of facilities to be built jointly

Instead of a regulator, the network operators decide cooperatively by bargaining the amount of facilities they build jointly, which we represent by I_s, after the network operators have decided their infrastructure levels. We are concerned about how the decision about I_s may influence on entry.

The game is the same first game as in the previous sections, of course, with the difference that the network operators decide their infrastructure levels sequentially instead of simultaneously.

At the last stage, when the network operators compete in prices, the equilibrium prices are the same than in the other scenarios studied up to now. Following backward induction, we continue solving the Bargaining Stage. As previously, we make use of the Nash Bargaining Solution:

$$\max_{I_s} \Pi_A^{\beta}\Pi_B^{1-\beta}$$

$$s.t.\ I_s \leq \min\{I_A, I_B\},\ \beta \in (0,1),$$

where

$$\Pi_A = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^{\alpha}$$

and

$$\Pi_B = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^\alpha$$

Note that, up to this stage, we are solving the same problem that when the network operators decided their infrastructure levels simultaneously. This means that the solution must be the same, namely $I_s = \min\{I_A, I_B\}$ or $I_s = 0$. To check so, we solve the first stage of the game where the network operators choose sequentially their infrastructure levels. We suppose that $\min\{I_A, I_B\} = I_B$, which is the more likely result in equilibrium. The entrant solves:

$$if\ I_s = 0\ \Pi_B^* = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^\alpha$$

or

$$if\ I_s = I_B\ \Pi_B^* = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^\alpha$$

The first order conditions are respectively:

$$if\ I_s = 0 \Rightarrow \frac{\partial \Pi_B^*}{\partial I_B}\frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha I_B^{\alpha-1} = 0$$

and

$$if\ I_s = I_B \Rightarrow \frac{\partial \Pi_B^*}{\partial I_B}\frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha(1-\gamma)^\alpha I_B^{\alpha-1} = 0$$

The incumbent solves:

$$\max_{I_A}\ \Pi_A^* = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^\alpha$$

$$s.t.\ I_s = 0,\ \frac{\partial \Pi_B^*}{\partial I_B}\frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha I_B^{\alpha-1} = 0$$

or

$$s.t.\ I_s = I_B,\ \frac{\partial \Pi_B^*}{\partial I_B}\frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha(1-\gamma)^\alpha I_B^{\alpha-1} = 0$$

We can see that the maximization problems are identical to the maximization problems when I_s is decided by a regulator. The only difference is that now the network operators can also choose $I_s = 0$, the disagreement point. Looking at the solutions of the problems we have computed before in this section, we can see that the entrant chooses the minimum amount of infrastructure level $\underline{I}$, under $I_s = 0$ and $I_s = I_B$. However, the infrastructure level chosen by the incumbent is higher under $I_s = I_B$ than under $I_s = 0$. This means that the infrastructure level gap is wider and therefore the revenue will be higher for the incumbent and lower for the entrant if $I_s = I_B$. The costs are lower for both of them. Then, it is rather clear that the incumbent always prefers $I_s = I_B$ to $I_s = 0$, but the entrant only prefers $I_s = I_B$ to $I_s = 0$ if σ is sufficiently large. In other words, the entrant prefers $I_s = I_B$ if the saved costs are higher than the lost revenues. Given this, we can conclude:

Proposition 13 *Let the network operators decide sequentially their infrastructure levels, let them decide cooperatively the amount of facilities they build and let σ to be sufficiently large. Then, the network operators choose the same amount of infrastructure level as when the amount of facilities they build jointly is decided by a regulator.*

The regulator and the network operators choose the same if σ is large. Otherwise, the entrant chooses the disagreement point, $I_s = 0$. In this case, it is needed the presence of a regulator, because it may happen that the network operators deviate from what the regulator would do.

About the robustness of this result with respect to the timing of the game, There may be two possibilities. First, network operators decide cooperatively the amount of facilities to build jointly and afterwards they choose their infrastructure level sequentially. Second, the two stages we have described in the first option are decided simultaneously, but it is very unlikely that these possibilities may arise in reality.

6 Conclusions

We have analyzed how the possibility of building jointly facilities affects the network operators decisions on network qualities. We have shown that the amount of facilities built jointly gives incentives to raise the infrastructure levels. In particular, when the network operators have to decide their infrastructure levels simultaneously (depending on the timing of the network operators decisions) both network operators choose higher infrastructure levels, independently of whether the amount to be built is set by a regulator or it is set cooperatively by the network operators. In this case, the possibility of building jointly facilities gives the same incentives to network operators and to the regulator. All of them see it as an infrastructure level cost reduction that makes "cheaper" for the network operators to invest in their network qualities.

When we analyze entry we find different results. If building facilities jointly is allowed the incumbent improves her infrastructure level and the entrant keeps choosing the same infrastructure level as if building jointly was not allowed. This is due to the fact that given any infrastructure level chosen by the entrant, the incumbent chooses her infrastructure level in such a way that the only profitable possibility for the entrant is to choose the lowest infrastructure level. This is so, because the incumbent is the first to choose the infrastructure level and she profits from his first mover advantage. The incumbent profits are always higher because he chooses a higher quality network compared to the entrant. The entrant enjoys lower revenues but at the same time lower costs. This comes from the fact that she chooses the cheapest infrastructure level, given that the network operators build jointly facilities. Thus, depending on the net sum of these two effects, the entrant may have lower profits. If the entrant has lower profit, she would choose the disagreement point for the bargaining stage and no facilities would be jointly built.

It is important to point out that the network operators decide the same network qualities when they decide first, about their network qualities and second, about the amount of facilities they build jointly as the quality jointly built is decided by a regulator. But, if the timing of the network operators is to decide, first about the amount of facilities they build jointly and second, about their network quality, the network operators choose very low network qualities which is very far from the ideal for social welfare.

National Regulatory Boards should allow network operators to build facilities jointly, but they have to control the timing of the network operators decisions. In doing so, they force them to build networks with the highest possible quality. In the case of entry, it may be necessary the presence of a regulator too. The network operators may decide not to build facilities jointly. It is interesting to point out that a regulator may be necessary not because the network operators may damage competition using the possibility of building facilities jointly but because they may decide to build jointly less than what is required from a social point of view.

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A appendix

A.1 Proof Proposition 1

Assume that $\min\{I_i, I_j\}$ is I_j and $I_i > I_j$. The social planner asks the network operator with $\min\{I_i, I_j\}$ for a higher infrastructure level to relax the constraint $s \leq \min\{I_i, I_j\}$, this would allow the network operators to build jointly more facilities, we would find out a situation where $I_j^P > I_i^P$, which is a contradiction. Then, the only possible solution is $I_j^P = I_i^P = I^P$.

We substitute $s = I_i$, in the objective function and by taking the derivative, we obtain that in equilibrium the optimum infrastructure levels hold the following condition:

$$I_i \in \Re_+ \; / \; \frac{1}{2}\frac{\partial u_i}{\partial I_i} - \alpha(1-\gamma)^\alpha I_i^{\alpha-1} = 0$$

A.2 Proof Proposition 2

The proof is straight forward. Note that the marginal costs of the infrastructure level decreases with I_s and the marginal revenues keep constant with I_s. This allows the network operators to choose higher infrastructure levels, which means a higher level of welfare, because final prices are constant.

A.3 Proof Proposition 3

If $I_s < I_i$, we take the derivative with respect to I_s of the first order condition. This gives us the equilibrium infrastructure level when the network operators are allowed to build jointly facilities:

$$\frac{\partial}{\partial I_s}(\frac{1}{3}\frac{\partial u_i}{\partial I_i} - \alpha(I_i - \gamma I_s)^{\alpha-1}) =$$

$$= \frac{1}{3}\frac{\partial}{\partial I_s}\frac{\partial u_i}{\partial I_i} - \frac{\partial}{\partial I_s}(\alpha(I_i - \gamma I_s)^{\alpha-1}) = \alpha\gamma(\alpha-1)(I_i - \gamma I_s)^{\alpha-1} > 0$$

Therefore the marginal cost of the infrastructure level is lower, the marginal revenue keeps constant and, thus, in equilibrium, the network operators infrastructure levels raise.

If $I_s = I_i$, the network operators profit functions are:

$$\Pi_i^* = \frac{1}{2\sigma} + \frac{u_i - u_j}{3} + \sigma(\frac{(u_i - u_j)^2}{18}) - (1-\gamma)^\alpha I_i^\alpha$$

If we take the derivative con respect to I_i to the profit function and we apply symmetry:

$$\frac{\partial \Pi_i^*}{\partial I_i} = \frac{1}{3}\frac{\partial u_i}{\partial I_i} - \alpha(1-\gamma)^\alpha I_i^{\alpha-1} = 0$$

If we compare this first order condition with the one we obtain when $I_s = 0$

$$\frac{\partial \Pi_i^*}{\partial I_i} = \frac{1}{3}\frac{\partial u_i}{\partial I_i} - \alpha I_i^{\alpha-1} = 0$$

we see that the marginal revenue keeps constant and the marginal cost is lower when $I_s = I_i$. Therefore, in equilibrium, the infrastructure levels are higher under $I_s = I_i$.

A.4 Proof Proposition 4

The social planner optimum infrastructure level satisfies:

$$\frac{1}{2}\frac{\partial u_i}{\partial I_i} - \alpha(1-\gamma)^\alpha I_i^{\alpha-1} = 0$$

and the network operators optimum infrastructure level satisfies:

$$\frac{1}{3}\frac{\partial u_i}{\partial I_i} - \alpha(1-\gamma)^\alpha I_i^{\alpha-1} = 0$$

As far as both marginal costs are identical, the proportion between the social planner and both network operators optima is the ratio between the marginal revenues

$$\frac{I^P}{I^*} = \frac{2}{3}$$

where I^P is the social planner optimum and I^* is the network operators optimum. This means that the network operators always choose lower infrastructure levels compared with what a social planner would do. Given this, as the network operators infrastructure levels raise with I_s, as we can deduce from proposition 3, and therefore, social welfare, the regulator always chooses the highest possible I_s, $I_s = I_i$.

A.5 Proof Proposition 5

Let $I_s = I_{I_i}$ and let the equilibrium investments be I_i^* without building jointly and I_i^{**} when building jointly. Then, the network operators enjoy higher profits if:

$$\Pi_i^* - \Pi_i^{**} = (\frac{1}{2\sigma} - (I_i^*)^\alpha) - (\frac{1}{2\sigma} - (1-\gamma)^\alpha I_i^{**\alpha}) \leq 0 \Rightarrow$$

$$\Rightarrow (1-\gamma)^\alpha I_i^{**\alpha} \leq (I_i^*)^\alpha$$

$$(1-\gamma) I_i^{**} \leq I_i^*$$

To know if last condition holds, we use the optimality conditions:

$$\frac{1}{3}\frac{\partial u_i}{\partial I_i}|_{I_i = I_i^*} = \alpha (I_i^*)^{\alpha-1}$$

and

$$\frac{1}{3}\frac{\partial u_i}{\partial I_i}|_{I_i = I_i^{**}} = \alpha (1-\gamma)^\alpha (I_i^{**})^{\alpha-1}$$

If the condition needed for a higher profit when the network operators are allowed to build jointly holds, then:

$$\frac{1}{3}\frac{\partial u_i}{\partial I_i}|_{I_i = I_i^{**}} = \alpha (1-\gamma)^\alpha I_i^{**\alpha-1} \leq$$

$$\leq \alpha (I_i^*)^{\alpha-1} = \frac{1}{3}\frac{\partial u_i}{\partial I_i}|_{I_i = I_i^*} \Rightarrow$$

$$\frac{1}{3}\frac{\partial u_i}{\partial I_i}_{I_i = I_i^{**}} \leq \frac{1}{3}\frac{\partial u_i}{\partial I_i}_{I_i = I_i^*}$$

As it is known from previous results, $I_i^* < I_i^{**}$, this together with the fact that u_i is concave yields $\frac{1}{3}\frac{\partial u_i}{\partial I_i}_{I_i = I_i^{**}} < \frac{1}{3}\frac{\partial u_i}{\partial I_i}_{I_i = I_i^*}$ and therefore $\Pi_i^* < \Pi_i^{**}$.

A.6 Proof Lemma 1

If we take the derivative with respect to the objective function we obtain:

$$\beta\frac{\partial\Pi_A}{\partial I_s}\Pi_A^{\beta-1}\Pi_B^{1-\beta}+(1-\beta)\frac{\partial\Pi_B}{\partial I_s}\Pi_A^{\beta}\Pi_B^{-\beta}$$

Working out the expression:

$$\beta\frac{\partial\Pi_A}{\partial I_s}(\frac{\Pi_B}{\Pi_A})^{(1-\beta)}+(1-\beta)\frac{\partial\Pi_B}{\partial I_s}(\frac{\Pi_A}{\Pi_B})^{\beta}$$

Notice that this expression is positive, because both derivatives, β, $1-\beta$ and the profit function are positives. Thus, the only possible solution to the problem is $I_s=\min\{I_A,I_B\}$ or $I_s=0$.

A.7 Proof Proposition 6

As we have seen in proposition 5, the network operators enjoy higher profits when they can build jointly facilities. Then, the possibility of $I_s=0$ is ruled out. Given this, the problem is identical to the problem we have solved when a regulator sets the quantity the network operators can build jointly. Therefore, given that we have two identical problems, we have two identical solutions.

A.8 Proof Proposition 7

In the game when, first, the network operators decide the amount of facilities that they build jointly and second, they choose their infrastructure levels, the equilibrium prices of the last stage are:

$$p_i=\frac{1}{\sigma}+\frac{(u_i-u_j)}{3}\ i=A,B\ j\neq i$$

Following backward induction, the network operators decide non-cooperatively their infrastructure levels:

$$\max_{I_i}\ \Pi_i^*=\frac{1}{2\sigma}+\frac{(u_i-u_j)}{3}+\sigma\frac{(u_i-u_j)^2}{18}-(I_i-\gamma I_s)^\alpha$$

$$s.t.\ I_i\geq I_s$$

Given the constraint, we may have two possible solutions, $I_i^* > I_s$ and $I_i = I_s$. Our first task is to prove that $I_i^* > I_s$ is not part of a SPNE. If it were part of it, the network operators would solve:

$$\max_{I_s} \ B = \Pi_A^{\beta}\Pi_B^{1-\beta}$$

$$s.t. \ \Pi_A = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^{\alpha}$$

and

$$\Pi_B = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^{\alpha}$$

The first order condition of the bargaining problem is:

$$\frac{\partial B}{\partial I_s} = \beta\frac{\partial \Pi_A}{\partial I_s}(\frac{\Pi_B}{\Pi_A})^{1-\beta} + (1-\beta)\frac{\partial \Pi_B}{\partial I_s}(\frac{\Pi_A}{\Pi_B})^{\beta} > 0$$

This implies that, in equilibrium, $I_s = I_i^*$, which is a contradiction. As we have seen previously, the equilibrium must be symmetric. $I_s = I_i^* \ for \ i = A, B$. If we bring this result to the bargaining problem, we get:

$$\max_{I_s} \ B = \Pi_A^{\beta}\Pi_B^{1-\beta}$$

$$s.t. \ \Pi_A = \frac{1}{2\sigma} - (1-\gamma)^{\alpha} I_s^{\alpha}$$

and

$$\Pi_B = \frac{1}{2\sigma} - (1-\gamma)^{\alpha} I_s^{\alpha}$$

The first order condition of the problem is:

$$\frac{\partial B}{\partial I_s} = \beta\frac{\partial \Pi_A}{\partial I_s}(\frac{\Pi_B}{\Pi_A})^{1-\beta} + (1-\beta)\frac{\partial \Pi_B}{\partial I_s}(\frac{\Pi_A}{\Pi_B})^{\beta} < 0$$

In equilibrium, the network operators infrastructure levels are $I_A^* = I_B^* = \underline{I}$. Moreover, the network operators build all the facilities jointly.

For the second game, where the network operators decide simultaneously the amount of facilities they build jointly and their infrastructure levels, the equilibrium prices of the last stage are:

$$p_i = \frac{1}{\sigma} + \frac{(u_i - u_j)}{3} \; for \; i = A, B \; and \; j \neq i$$

We solve the second stage. We start solving the problem where the network operators decide cooperatively the amount they build jointly:

$$\max_{I_s} \; B = \Pi_A^{\beta} \Pi_B^{1-\beta}$$

$$s.t. \; I_s \leq \min\{I_A, I_B\}, \;\; \beta \in (0,1)$$

where

$$\Pi_A = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma \frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^{\alpha}$$

and

$$\Pi_B = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma \frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^{\alpha}$$

The first order condition is:

$$\frac{\partial B}{\partial I_s} = \beta (\frac{\Pi_B}{\Pi_A})^{1-\beta} \frac{\partial \Pi_A}{\partial I_s} + (1-\beta)(\frac{\Pi_A}{\Pi_B})^{\beta} \frac{\partial \Pi_B}{\partial I_s}$$

As

$$\frac{\partial \Pi_A}{\partial I_s} = \alpha \gamma (I_A - \gamma I_s)^{\alpha - 1} > 0$$

and

$$\frac{\partial \Pi_B}{\partial I_s} = \alpha \gamma (I_B - \gamma I_s)^{\alpha - 1} > 0$$

The solution is $I_s = \min\{I_A, I_B\}$. Therefore, the network operators solve these problems in order to know their infrastructure levels:

$$\max_{I_i} \; \Pi_i^* = \frac{1}{2\sigma} + \frac{(u_i - u_j)}{3} + \sigma \frac{(u_i - u_j)^2}{18} - (1-\gamma)^{\alpha} I_i^{\alpha}$$

In equilibrium, the following first order conditions hold:

$$\frac{1}{3} \frac{\partial u_i}{\partial I_i} - \alpha (1-\gamma)^{\alpha} I_i^{\alpha - 1} = 0$$

Then, the solution to this second game coincidence with the solution to the former game in the paper.

A.9 Proof Lemma 2

As $u_A > u_B \; \forall I_A = I_B$ and $u_i = 0$ if $I_i = 0$ and concavity of u_A and u_B imply $\frac{\partial u_A}{\partial I_A} > \frac{\partial u_B}{\partial I_B}$. If we work out the first order conditions, we obtain:

$$\frac{1}{3} + \frac{\sigma}{9}(u_A - u_B) = \frac{\alpha I_A^{\alpha-1}}{\frac{\partial u_A}{\partial I_A}}$$

$$\frac{1}{3} + \frac{\sigma}{9}(u_B - u_A) = \frac{\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}}$$

If we subtract the second equation to the first one:

$$\frac{2\sigma}{9}(u_A - u_B) = \frac{\alpha I_A^{\alpha-1}}{\frac{\partial u_A}{\partial I_A}} - \frac{\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}}$$

and finally:

$$\frac{\alpha I_A^{\alpha-1}}{\frac{\partial u_A}{\partial I_A}} = \frac{\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}} + \frac{2\sigma}{9}(u_A - u_B)$$

From this equation and from the conditions $u_A > u_B$, $\frac{\partial u_A}{\partial I_A} > \frac{\partial u_B}{\partial I_B}$ and the characteristics of the cost functions, we can conclude that in equilibrium, the network operator A infrastructure level is higher, $I_A^* > I_B^*$.

A.10 Proof Proposition 8

The regulator always chooses $I_s = I_B$. Given this, the derivative with respect to I_s of the network operator A first order condition of the profit maximization problem is:

$$\frac{\partial}{\partial I_B}\frac{\partial \Pi_A^*}{\partial I_A} = \alpha(\alpha-1)\gamma(I_A - \gamma I_B)^{(\alpha-2)} > 0$$

To know how the first order condition changes when $I_s = I_i$ respect to the case when $I_s = 0$, we subtract one to the other one:

$$\frac{\partial \Pi_B^*}{\partial I_B}(I_s = I_B) - \frac{\partial \Pi_B^*}{\partial I_B}(I_s = 0) = \alpha I_B^{\alpha-1} - \alpha(1-\gamma)^{\alpha} I_B^{\alpha-1} > 0 \Rightarrow 1-(1-\gamma)^{\alpha} > 0$$

which is true because $\gamma < 1$ and $\alpha > 1$. Therefore, as both first order conditions shift upwards, both infrastructure levels are higher, $I_i^{**}(I_s = I_B) > I_i^*(I_s = 0),\; i = A, B$.

A.11 Proof Proposition 9

We know that $I_s = I_B$. Recall the first order conditions:

$$\frac{1}{3}\frac{\partial u_A}{\partial I_A} + \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_A}{\partial I_A} = \alpha(I_A - \gamma I_B)^{\alpha-1}$$

and

$$\frac{1}{3}\frac{\partial u_B}{\partial I_B} + \frac{\sigma}{9}(u_B - u_A)\frac{\partial u_B}{\partial I_B} = \alpha(1-\gamma)^{\alpha} I_B^{\alpha-1}$$

If we work out the expression, we obtain that in equilibrium:

$$I_A^{**} = \left(\frac{\frac{1}{3}\frac{\partial u_A}{\partial I_A} + \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_A}{\partial I_A}}{\alpha}\right)\Big|_{I_A = I_A^{**}}^{\frac{1}{(\alpha-1)}} + \gamma I_B^{**}$$

$$I_B^{**} = \left(\frac{\frac{1}{3}\frac{\partial u_B}{\partial I_B} + \frac{\sigma}{9}(u_B - u_A)\frac{\partial u_B}{\partial I_B}}{\alpha(1-\gamma)^{\alpha}}\right)\Big|_{I_B = I_B^{**}}^{\frac{1}{(\alpha-1)}}$$

If we subtract the second to the first equation and we work out the expression:

$$I_A^{**} - I_B^{**} > I_A^{*} + \gamma I_B^{**} - I_B^{**} - \frac{1-(1-\gamma)^{\frac{\alpha}{\alpha-1}}}{(1-\gamma)^{\frac{\alpha}{\alpha-1}}} I_B^{**} = I_A^{*} - I_B^{*}$$

where I_A^* and I_B^* are the network operator A and B infrastructure levels when they are not allowed to build jointly facilities. Therefore, as $\alpha \geq 2$ $I_A^{**}(I_s = I_B) - I_B^{**}(I_s = I_B) < I_A^{*}(I_s = 0) - I_B^{**}(I_s = 0)$

A.12 Proof Proposition 10

From proposition 9, we know that $I_A^* - I_B^* > I_A^{**} - I_B^{**}$ $\forall \alpha \geq 2$. This implies that $(u_A - u_B)|_{I_s=0} = u_A^* - u_B^* > (u_A - u_B)|_{I_s=I_B} = u_A^{**} - u_B^{**}$. This means that the network operator A revenues decreases and the network operator B increases. At the same time, both network operators enjoy lower infrastructure level costs. Then, it is clear that the network operator B gets higher profits under $I_s = I_B$. Therefore, he never chooses the disagreement point, $I_s = 0$. The network operator also gets higher profits, because the cost reduction is higher than the revenue reduction. This is so because the cost function is convex and the utility function is concave. Concavity of the utility function implies that, given the difference in the infrastructure levels when $I_s = 0$ and when $I_s = I_B$, although the infrastructure level gap is narrower, the revenue reduction is very small, because the marginal utility is decreasing. Moreover, the utility function does not change with I_s. On the other hand, as the cost function is convex, the marginal cost is increasing and the cost reduction $I_s = I_B$ has a big impact in the costs because they are very much reduced, much more than the revenues.

A.13 Proof Proposition 11

Network operators problems when they decide non-cooperatively their infrastructure levels are, respectively:

$$\max_{I_A} \Pi_A^* = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^\alpha$$

$$s.t.\ I_A \geq I_s,\ u_A > u_B\ \forall I_A = I_B$$

and

$$\max_{I_B} \Pi_B^* = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^\alpha$$

$$s.t.\ I_B \geq I_s\ u_A > u_B\ \forall I_A = I_B$$

Given the constraints, we have two possible solutions, $I_B^* > I_s$ and $I_B^* = I_s$. I_A is always higher than I_B because $u_A > u_B\ \forall I_A = I_B$. First, we suppose that, in equilibrium, $I_i^* > I_s$. Then, the network operators solve:

$$\max_{I_s} B = \Pi_A^\beta \Pi_B^{1-\beta}$$

$$s.t.\ \Pi_A = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^\alpha$$

$$\Pi_B = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^\alpha$$

$$u_A > u_B \; \forall I_A = I_B$$

The first order condition of the problem is:

$$\frac{\partial B}{\partial I_s} = \beta \frac{\partial \Pi_A}{\partial I_s} (\frac{\Pi_B}{\Pi_A})^{1-\beta} + (1-\beta) \frac{\partial \Pi_B}{\partial I_s} (\frac{\Pi_A}{\Pi_B})^{\beta} > 0$$

This implies that, in equilibrium $I_B^* = I_s$, which is a contradiction. Next, we see what happens when $I_B = I_s$. First, we check the level of investment chosen by the network operator A. This comes from the following expression:

$$\frac{\partial \Pi_A^*}{\partial I_A} = \frac{1}{3} \frac{\partial u_A}{\partial I_A} + \frac{\sigma}{9} (u_A - u_B) \frac{\partial u_A}{\partial I_A} - \alpha (I_A - \gamma I_s)^{\alpha - 1} = 0$$

We also need to know about the bargaining problem:

$$\max_{I_s} \; B = \Pi_A^{\beta} \Pi_B^{1-\beta}$$

$$s.t. \; \Pi_A = \frac{1}{2\sigma} + \frac{(u_A - u_B(I_s))}{3} + \sigma \frac{(u_A - u_B(I_s))^2}{18} - (I_A - \gamma I_s)^{\alpha}$$

and

$$\Pi_B = \frac{1}{2\sigma} + \frac{(u_B(I_s) - u_A)}{3} + \sigma \frac{(u_A - u_B(I_s))^2}{18} - (I_B - \gamma I_s)^{\alpha}$$

If we take the first order conditions, we obtain:

$$\frac{\partial \Pi_A}{\partial I_s} = \beta \frac{\partial \Pi_A}{\partial I_s} (\frac{\Pi_B}{\Pi_A})^{1-\beta} + (1-\beta) \frac{\partial \Pi_B}{\partial I_s} (\frac{\Pi_A}{\Pi_B})^{\beta} = 0$$

where

$$\frac{\partial \Pi_A}{\partial I_s} = -\frac{1}{3} \frac{u_B(I_s)}{\partial I_s} - \frac{\sigma}{9} (u_A - u_B(I_s)) \frac{\partial u_B(I_s)}{\partial I_s} + (I_A - \gamma I_s)^{\alpha - 1}$$

This expression has got only a positive term. This means that the network operator A wants I_s very low and it may be so low that it could be lower than the minimum requirement. The network operator B wants I_s as high as it is possible. When the timing of the game is, first the network operators decide

I_A, I_B and, second, they bargain over I_s, as $\beta \in (0,1)$, the equilibrium I_B^{**} is lower than in the game where first network operators decide I_A and I_B and second, they bargain over I_s. If we plug this result in the network operator A first order condition we can see that the network operator A infrastructure level, I_A^{**}, is also lower than in the game with different timing.

Assume that network operators decide simultaneously about the amount of facilities they build jointly, I_s and the infrastructure levels, I_A and I_B. We solve the second stage. We start solving the problem where the network operators decide cooperatively the amount they build jointly:

$$\max_{I_s} \ B = \Pi_A^{\beta} \Pi_B^{1-\beta}$$

$$s.t.\ I_s \leq \min\{I_A, I_B\},\ \beta \in (0,1)\ and\ u_A > u_B\ \forall I_A = I_B$$

where

$$\Pi_A = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_s)^{\alpha}$$

and

$$\Pi_B = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_B - \gamma I_s)^{\alpha}$$

The first order condition is:

$$\frac{\partial B}{\partial I_s} = \beta(\frac{\Pi_B}{\Pi_A})^{1-\beta}\frac{\partial \Pi_A}{\partial I_s} + (1-\beta)(\frac{\Pi_A}{\Pi_B})^{\beta}\frac{\partial \Pi_B}{\partial I_s}$$

As

$$\frac{\partial \Pi_A}{\partial I_s} = \alpha\gamma(I_A - \gamma I_s)^{\alpha-1} > 0$$

and

$$\frac{\partial \Pi_B}{\partial I_s} = \alpha\gamma(I_B - \gamma I_s)^{\alpha-1} > 0$$

The solution is $I_s = \min\{I_A, I_B\}$. Therefore, the network operators solve these problems in order to know their infrastructure levels. For the network operator B:

$$\max_{I_B}\ \Pi_B^* = \frac{1}{2\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (1-\gamma)^\alpha I_B^\alpha$$

In equilibrium, the following first order holds:

$$\frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha(1-\gamma)^\alpha I_B^{\alpha-1} = 0$$

For the network operator A:

$$\max_{I_A}\ \Pi_A^* = \frac{1}{2\sigma} + \frac{(u_A - u_B)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (I_A - \gamma I_B)^\alpha$$

In equilibrium, the following first order holds:

$$\frac{1}{3}\frac{\partial u_A}{\partial I_A} + \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_A}{\partial I_A} - \alpha(I_A - \gamma I_B)^{\alpha-1} = 0$$

Then, the solution to this second game coincidence with the former game in the paper.

A.14 Proof Lemma 3

If we take the incumbent maximization problem constraint and we work it out we obtain:

$$u_A - u_B = \left(\frac{1}{3} - \frac{\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}}\right)\frac{9}{\sigma}$$

If we substitute the expression in the incumbent maximization problem:

$$\max_{I_A}\ \frac{1}{2\sigma} + \left(\frac{1}{3} - \frac{\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}}\right)\frac{3}{\sigma} + \left(\frac{1}{3} - \frac{\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}}\right)^2\frac{9}{2\sigma} - I_A^\alpha$$

We take the first order condition and we see that is negative:

$$\frac{\partial}{\partial I_A} = -\alpha I_A^{\alpha-1} < 0$$

The incumbent chooses the quantity that holds his constraint. We have multiple equilibria. Any pair (I_A, I_B) that satisfies the incumbent constraint is an equilibrium. If we check the restriction it is easy to see that the incumbent always chooses a higher infrastructure level than the entrant. Given the multiple equilibria, the more likely equilibrium is the one where the entrant chooses the minimum infrastructure level that network operators have to supply. This is because, in this equilibrium, both network operators are better off. Given the entrant investment, the incumbent chooses the investment that comes out from its maximization constraint and it is easy to see that is higher than the entrant infrastructure level.

A.15 Proof Proposition 12

If we work out the incumbent maximization problem constraint, we obtain:

$$(u_A - u_B) = (\frac{1}{3} - \frac{\alpha(1-\gamma)^\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}})\frac{9}{\sigma}$$

If we substitute this expression in the incumbent objective function, the maximization problem becomes:

$$\max_{I_A} \frac{1}{2\sigma} + (\frac{1}{3} - \frac{\alpha(1-\gamma)^\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}})\frac{3}{\sigma} + (\frac{1}{3} - \frac{\alpha(1-\gamma)^\alpha I_B^{\alpha-1}}{\frac{\partial u_B}{\partial I_B}})^2\frac{9}{2\sigma} - (I_A - \gamma I_B)^\alpha$$

If we take the first order condition we see that is negative:

$$\frac{\partial}{\partial I_A} = -\alpha(I_A - \gamma I_B)^{\alpha-1} < 0$$

The incumbent chooses the least possible quantity that holds its constraint. Again, we have multiple equilibria, any pair (I_A, I_B) that holds the incumbent constraint is an equilibrium and if we check the restriction it is easy to see that the incumbent always chooses a higher infrastructure level more than the entrant. Given the multiple equilibria, the more likely equilibrium is the one where the entrant chooses the minimum investment level that network operators have to supply. This is because in this equilibrium both network operators are better off. So the entrant repeats the same investment than when building jointly was not allowed. The incumbent investment comes out again from its maximization constraints and it is higher. We prove it just comparing the constraint under $I_s = 0$ and $I_s = I_B$.

$$If\ I_s = 0,\ then\ \frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha I_B^{\alpha-1} = 0$$

$$if\ I_s = I_B\ then\ \frac{1}{3}\frac{\partial u_B}{\partial I_B} - \frac{\sigma}{9}(u_A - u_B)\frac{\partial u_B}{\partial I_B} - \alpha(1-\gamma)^\alpha I_B^{\alpha-1} = 0$$

If we subtract the first equation to the second equation, we obtain:

$$\alpha(1-\gamma)^\alpha I_B^{\alpha-1} - \alpha I_B^{\alpha-1} \Rightarrow (1-\gamma)^\alpha - 1 < 0$$

Therefore, the constraint is relaxed and that allows the incumbent to choose a higher infrastructure level.

A.16 Proof Proposition 13

When $I_s = 0$, the network operator B profit function is:

$$\Pi_B^*(I_s = 0) = \frac{1}{\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - I_B^\alpha$$

and when $I_s = I_B$, the network operator B profit function is:

$$\Pi_B^*(I_s = I_B) = \frac{1}{\sigma} + \frac{(u_B - u_A)}{3} + \sigma\frac{(u_A - u_B)^2}{18} - (1-\gamma)^\alpha I_B^\alpha$$

In equilibrium, $I_B^*(I_s = 0) = I_B^{**} = \underline{I}$. On the contrary, the network operator A chooses higher infrastructure level under $I_s = I_B$ than under $I_s = 0$. This implies that $(u_A - u_B)|_{I_s=0} = u_A^* - u_B^* < (u_A - u_B)|_{I_s=I_B} = u_A^{**} - u_B^{**}$, because the infrastructure level gap is wider when $I_s = I_B$. If we compare the profit under $I_s = 0$ and $I_s = I_B$, we see that:

$$\Pi_B^*(I_s = I_B) - \Pi_B^*(I_s = 0) = (\frac{1}{2\sigma} + \frac{u_B^* - u_A^*}{3} + \sigma\frac{(u_B^* - u_A^*)^2}{18} - \underline{I}^\alpha) -$$

$$-(\frac{1}{2\sigma} + \frac{u_B^* - u_A^*}{3} + \sigma\frac{(u_B^* - u_A^*)^2}{18} - (1-\gamma)^\alpha\underline{I}^\alpha) =$$

$$= \sigma(\frac{(u_A^{**} - u_B^{**})^2}{18} - \frac{(u_A^* - u_B^*)^2}{18}) + (1-(1-\gamma)^\alpha)\underline{I}^\alpha -$$

$$-(\frac{(u_A^{**} - u_B^{**})}{3} - \frac{(u_A^* - u_B^*)}{3})$$

If we want $\Pi_B^*(I_s = I_B) - \Pi_B^*(I_s = 0)$ to be positive, we need σ to be sufficiently large to overcome the negative term $(\frac{(u_A^{**} - u_B^{**})}{3} - \frac{(u_A^* - u_B^*)}{3})$.