UNIVERSIDAD CARLOS III DE MADRID working papers

Working paper 4-28
Economics Series 09
May 2004

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624 9875

DOES ASYMMETRIC INFORMATION PROMOTE TALENTED PEOPLE? *

Ana Hidalgo-Cabrillana [1]

Abstract

The existing literature regarding issues of imperfect capital markets in connection with intergenerational mobility recognizes that imperfections in the capital markets represent a barrier to intergenerational mobility. This paper argues against this general thought. Contrary to this opinion, our model shows that when banks do not know the ability of the borrower, they respond to this asymmetry of information by devoting higher loan to talented borrowers. A force that helps poor and talented individuals to become educated and to catch up with the rich ones.

Keywords: Intergenerational mobility, Inequality, Capital markets imperfections, Adverse selection.

1. Departamento de Economía, Universidad Carlos III de Madrid, E-mail: ahidalgo@eco.uc3m.es

* I am indebted to Andrés Erosa and José-Vicente Rodríguez Mora for helpful discussion and comments. I am thankful for the comments of Belén Jerez, and participants at the Seventh Workshop on Dynamic Macroeconomics. Financial support from "Generalitat the Catalunya: Beques predoctoral per la formaciò de personal investigador (FI/FIAP)" is gratefully acknowledge. All errors are mine own.

1 Introduction

Do capital market imperfections (CMI for short) increase inequality and intergenerational mobility? Are CMI a barrier to mobility? A huge literature has been devoted to the implication of asymmetries of information on the distribution of income and intergenerational mobility (Becker and Tomes (1986), Maoz and Moav (1999), Mulligan (1996), Loury (1981), Owen and Weil (1998)). These papers tell us that under CMI, inequality becomes persistent since poor individuals do not have access to the same investment opportunities available for the rich. Therefore, CMI harm the poor, and more specifically lead to higher inequality and lower mobility.

This line of research has received such widespread support that a recent survey of this work concludes: "[...] persistence of inequalities across generations is possible only if capital markets are imperfect" (Aghion and Bolton, (1992) p. 606). Certainly, both the economic and public policy literatures have taken this argument for granted and based on this they have developed different policy analyses (public education, education subsidies or taxation programs for example).

A careful reading of this literature suggests that most of the existing research in this area has been developed under the assumption that capital market imperfections are "exogenous". Under exogenous CMI the credit limit is fixed and independent of the observable characteristics and decisions of individuals. Therefore, by exogenous CMI we refer to situations where credit constraints among poor people are introduced without providing any micro-foundations. In some cases, for example, this credit constraint is taken to the extreme such that agents can not borrow.[1] Exogenous credit constraints are sometimes presented by an exogenous and substantial wedge between the cost of borrowing and the return on lending.[2]

In contrast to these analyses, this paper argues that when we endogenize CMI, intergenerational mobility may be promoted among poor and talented agents. In particular, recognizing that modern financial markets are characterized by a wide variety of informational imperfections, we endogenize capital market imperfections by assuming an adverse selection problem between borrowers and banks.[3] To this end, we develop a growth model where agents are heterogeneous in terms of inherited wealth as well as ability. There are two types of agents; low ability agents and high ability ones. Young agents can undergo private education, and the investment in human capital, which is divisible, may be financed by a loan market.[4] Even though banks know the inherited wealth

[1] For example, Becker and Tomes (1986), Mulligan (1996), Loury (1981), Owen and Weil (1998), Maoz and Moav (1999).

[2] See for example Galor and Zeira (1993).

[3] In adverse selection models, the existence of equilibrium is an important issue. With perfect competition among banks, existence is not ensured. See Rothschild and Stiglitz (1976) for an illustration of this problem.

[4] An example of the importance of the amount borrowed for education is the case of American law students. With rising tuition fees, students have borrowed more to pay for their education. The sums that students are borrowing are much larger today than they were ten

of each applicant they do not know the borrower's ability. In our model the returns from the investment in education are random. Our central assumption is that ability hjas a positive effect not only the success probability, which is also the probability of repaying the debt, but also the return from the investment in education.[5]

When banks cannot identify borrowers' ability, they offer a menu of contracts that satisfy the self-selection mechanism. In equilibrium, banks differentiate between agents by forcing talented borrowers to make an investment in human capital larger than they would have done in the first-best world. In this way low ability individuals do not pose as high ability ones. In equilibrium, high ability children from poor families get educated even more than they wish, so that both income mobility and human capital accumulation are larger than in the first best world.

The related literature can be classified into two different branches. The first one is when CMI are exogenous and the second when they are endogenous.

Most papers analyze exogenous CMI. This literature does not model the reasons for the imperfections. Their underlying conclusion is that imperfections in the capital markets represent a barrier to intergenerational mobility. The intuition behind this result is that when borrowing is expensive, individuals with low wealth have no longer access to the same investment opportunities as individuals with high wealth. In this context, inequality becomes persistent and intergenerational mobility decreases.

This literature assumes that borrowers and external suppliers of funds have the same information about the borrower's choice, investment opportunities, riskiness of projects, and output or profits. In practice, borrowers have significantly better information than outside investors about most aspects of the borrower's investment and its returns. For that reason, the second branch of the literature endogenizes CMI by assuming asymmetries of information. To the best of my knowledge, such literature typically uses a moral hazard framework (see Aghion and Bolton (1997), Piketty (1997), Banerjee and Newman (1991), (1993)). None of these papers, however, introduce adverse selection in the capital market to analyze intergenerational mobility.[6]

Aghion and Bolton (1997) and Piketty (1997) examine the interaction be-

years ago, even after adjusting for the cost of living. For graduates at many schools, cumulative debts of $40,000 from college and law school have become the norm (see Chambers (1994)).

[5] We assume that agents investing more in human capital receive a better quality of education. This helps the student's outcomes, so that he is more likely to succeed, work in what he was educated in, and earn a higher income than if he fails and becomes uneducated. The idea behind this assumption is that buying more education is equivalent to having a higher probability of finishing studies and becoming educated.

[6] There is a good reason to be interested in the adverse selection problem. In my model, adverse selection implies that the borrower knows the expected return and the risk of his investment project, whereas the bank knows only the expected return and the risk of the average investment project in the economy, and thus there may be no objective way to determine the likelihood of the loan repayment. In our credit markets the *promised* repayments on loans differ from the *actual* ones because of the uncertainty concerning the borrower's ability, namely the quality of the investment. This creates the risk of borrower default.

tween wealth distribution and the equilibrium interest rate. Our model differs from these, both in structure and results. Even though both papers study inequality, Piketty (1997) does not explicitly model mobility. In particular, his model assumes that individuals do not differ in ability, and steady state mobility is random and independent of abilities. In contrast, in our paper, mobility is a result of individuals' choices given their ex-ante heterogeneity and banks' decisions. Another important difference is that poor agents are credit rationed in their models, whereas in our model the opposite occurs since there is overinvestment among high-ability borrowers. The reason is simple. In their papers risk neutral agents invest in their own projects. Agents with low internal equity go to the capital markets to get into debt. The expected returns on the investments depend positively on the effort that agents supply, but effort is not observable by the bank. The more the agent has to borrow, the higher are the marginal returns to share with banks and consequently the less effort the agent supplies. Since poor agents have no incentives to supply too much effort, banks will react by rationing them.[7] Thus, in this type of model CMI leads persistent inequality.[8]

Our result, as we argue in more details in the next sections, is the opposite. Accordingly, this paper contributes to the theoretical understanding of the linkage between intergenerational mobility and CMI. Our results should be taken as a complement to existing studies, not only raising doubts about the "consistent message," but also suggesting that further careful reassessment of the interaction between CMI and income distribution needs to be considered.

The paper is organized as follows. We set up the model in Section2. The equilibrium in the capital market is described in Section 3. The consequences of asymmetric information in terms of mobility, inequality and education are developed in Section 4. Section 5 concludes. Finally, an appendix contains all omitted proofs.

2 The Economy

The economy is populated by two-period-lived overlapping generations of agents. When individuals are young, they receive an inherited transfer from parents and the ability shock is realized. Then, they make their economic decision whether

[7] Similarly, Banerjee and Newman (1993) show that imperfect capital markets and endogenously determined wages could be a source of persistent inequality. In their model, moral-hazard problems prevent poor agents from investing in large projects. However, Banerjee and Newman (1991) assume an utility function unbounded below with respect to consumption. This is equivalent to assume infinite liability, which implies that there is no credit rationing at all.

[8] Aghion and Bolton (1997) focus on finding conditions under which there is a *unique* steady state distribution of wealth. This condition is given by assuming rapid capital accumulation, that is, total absence of credit rationing. In Piketty (1997), *multiple* stationary interest rates and wealth distributions can exist because higher initial rates are self-reinforcing through higher credit rationing and lower capital accumulation.

to go to school or not (and how much to invest in schooling).[9] Schooling is costly since it requires a cost which is privately provided. There exists a loan market to get into debt if necessary and it is characterized by an adverse selection problem. The capital market is competitive.

2.1 Individuals and Human Capital Technology

The economy is populated by a continuum of families, indexed by $j \in (0,1]$. For simplicity, there is one member of each family born in each period t, so that there is no population growth; the parent-child connection creates a dynasty. Individuals differ in the initial wealth inherited from their parents and in their ability.

Let θ denote an agent's ability. Individuals can be either low ability, denoted by $\underline{\theta}$ type, or high ability, denoted by $\overline{\theta}$, where $\overline{\theta} > \underline{\theta} > 0$. The proportion of low ability is γ and that of high ability is $1-\gamma$, with $\gamma \in (0,1)$. We assume that agents know their own ability while banks know only the proportion of individuals of each type, as well as the inherited bequest of each applicant.

Individuals are risk neutral. When young, agents maximize utility which depends on the second period consumption, c_{t+1},and on the bequest given to their child, b_{t+1}. More specifically, the utility function takes the form $U_t = zc_{t+1}^{1-\alpha}b_{t+1}^{\alpha}$, where $z = (1-\alpha)^{\alpha-1}\alpha^{-\alpha}$. According to this utility function, agents allocate the final wealth between consumption, $c_{t+1} = (1-\alpha)y_{t+1}$, and transfers to his child, $b_{t+1} = \alpha y_{t+1}$. Hence, the indirect utility function is simply a linear function of the wealth realization, $V_t = y_{t+1}$.

The human capital technology, which is given by the function $h(\theta, I_t)$, is stochastic at the individual level. In particular, human capital can take two different values depending on the realization of idiosyncratic shocks. We assume that in case of success agents become educated and earn a high return, whereas in case of failure agents drop out from college and earn a low return as uneducated agents.[10] Banks are able to observe ex post, and without any cost, whether the investment in human capital fails or succeeds. Hence, the returns from the investment are given by the function,

$$h_{t+1}^{j} = h(\theta, I_t) = \begin{cases} h^e G_\theta & if\ she\ succeeds\ with \quad p(\theta, I) \\ h^u & if\ she\ fails\ with \quad 1-p(\theta, I), \end{cases} \tag{1}$$

where $\theta = \{\underline{\theta}, \overline{\theta}\}$ and I is the investment in education which takes values in the interval $[0, \infty)$. As we will see later on, since the amount of investment is divisible it can be used to convey information about the borrower's ability. The returns from the investment in human capital are such that educated agents accumulate higher human capital than uneducated agents, since $h^e > h^u \geq 0$ and $G_\theta > 1$ holds. Notice that the human capital of educated agents is affected

[9] Since primary education is generally publically provided, education in the context of this model is better interpreted as post-secondary education. For brevity, we refer to post-secondary education as educated and someone who has a failed investment in education or decides not go to the school as uneducated.

[10] The results remain unaffected even if we assume $h^u = 0$.

by talent through G_θ. Talented agents obtain more human capital since ($G_{\overline{\theta}} > G_{\underline{\theta}}$). Moreover, a talented agent, by definition, will succeed more frequently so $p(\overline{\theta}, I) > p(\underline{\theta}, I)$ is assumed.[11] Therefore, ability affects the returns from the investment as well as the probability of success.

Ability is not the sole determinant of the success probability. Investment in education is the other factor that influences it. More human capital investment results in a higher success probability but at a decreasing rate, so that $p_{II} < 0$. Moreover, talented agents have higher marginal returns in the successful state. We formalize this reasoning with the assumption that ability and the amount of investment are complementary factors in the production of human capital, so it holds that $p_{\theta I} > 0$. Thus, high ability people have higher total and marginal returns in the successful state. These assumptions are common in the human capital literature (e.g. Mulligan (1992)). More specifically, we use the probability function

$$p(\theta, I) = B(\theta)(1 - e^{-I}) \quad \text{and} \quad 1 - p(\theta, I) = 1 - B(\theta)(1 - e^{-I}), \tag{A1}$$

where $0 < B(\underline{\theta}) < B(\overline{\theta}) < 1$.

Note that if ability only affected the returns, and probability depended just on I, we would find that the full information contract is incentive compatible at any level of inherited wealth. On the other hand, if ability affected only the probability of success, the single crossing property would not be satisfied. Therefore we choose the formulation described in (1). Moreover, we have to impose the following restriction on the exogenous parameters:

$$\frac{a}{d} < 1 + \frac{aB(\underline{\theta})}{dB(\overline{\theta})} \ln(\frac{a}{d}), \tag{A2}$$

where $a = B(\overline{\theta})(h^e G_{\overline{\theta}} - h^u)$, and $d = B(\underline{\theta})(h^e G_{\underline{\theta}} - h^u)$. Assumption (A2) represents a necessary and sufficient condition to ensure that the full information contract is not incentive compatible at any level of inherited wealth among borrowers.

Agents live for two periods. In the first period, individuals learn their ability and receive an inherited wealth. The parental gift b_t received by each individual is publically known. This inherited wealth can be used either to finance education, since education is privately provided, or to invest in the capital market at the riskless interest rate R.[12] Given the properties of the investment probability, it is always profitable to invest in education.[13]

In our model, only a fraction of agents will borrow to finance their college investment from a bank if necessary. Banks offer contracts that we denote by

[11] This is consistent with the empirical papers of Manski and Wise (1983) and Stinebrickener and Stinebrickener (2001), which show that children with less acquired ability are more likely to drop out from college.

[12] Note that even though we have confined our analysis to investment in education, we can also apply this model to investment projects for potential entrepreneurs.

[13] The reason is that $lim_{I \to 0} p_I(\theta, I)(h^e G_\theta - h^u) > R$, which implies that $B(\theta)(h^e G_\theta - h^u) > R$ for any $\theta \in \{\underline{\theta}, \overline{\theta}\}$.

$\xi = (F_{t+1}, I_t)$, where F_{t+1} is the interest rate charged and I_t is the amount of investment in education. Some other agents have enough inherited wealth to finance their investment in education and they thus become lenders. That is, they decide to optimally invest the excess bequest in the capital market at the riskless rate of return (see the subsection First Best Investment below). In summary, individuals can either lend, borrow or not participate in the capital market. If they lend ($b_t^j > I$) the second period wealth is

$$y_{t+1}^j = \begin{cases} h^e G_\theta + R(b_t^j - I) & with \quad p(\theta, I) \\ h^u + R(b_t^j - I) & with \quad 1 - p(\theta, I), \end{cases}$$

where R is the opportunity cost of funds.

If they borrow ($b_t^j < I$) when young, their wealth is

$$y_{t+1}^j = \begin{cases} h^e G_\theta - F(I - b_t^j) & with \quad p(\theta, I) \\ h^u & with \quad 1 - p(\theta, I). \end{cases}$$

We are assuming limited liability, so that when projects succeed agents become educated and earn an income high enough to repay the debt. By contrast, when projects fail borrowers are unable to repay the debt.

If individuals do not participate in the capital market they invest their inherited bequest, so their second period income is

$$y_{t+1}^j = \begin{cases} h^e G_\theta & with \quad p(\theta, b_t^j) \\ h^u & with \quad 1 - p(\theta, b_t^j). \end{cases}$$

We will see below that in equilibrium only agents endowed with a bequest equal to the first best amount of investment become self-financed. All the others will be either lenders or borrowers.

At the beginning of the second period of their life (when they are old), the uncertainty regarding the investment is resolved. Afterwards, banks receive profits and agents obtain their income, which is allocated between consumption, $(1 - \alpha)y_{t+1}$, and transfers to the children, αy_{t+1}.

First Best Investment The first-best level of investment, which is denoted by I^*, maximizes the expected returns net of the opportunity cost of the investment,

$$I^* \equiv \arg\max_{I \geq 0} \; p(\theta, I)h^e G_\theta + (1 - p(\theta, I))h^u - RI.$$

The FOC is,

$$\frac{dp(\theta, I)}{dI}(h^e G_\theta - h^u) = R. \tag{2}$$

It is worth noting that the fundamental problem of the agent is to optimally decide how much of the inherited wealth is invested in human capital and how much is invested in the capital market. Equation (2) represents the non-arbitrage condition between human and physical capital. It tells us that the current gross interest rate R equals the expected marginal profit of the investment in human capital. From (2) and A1 (which provides the functional form of $p(\theta, I)$) we can derive the first best level of investment,

$$I_{\theta}^{*} = \ln\left(\frac{B(\theta)(h^{e}G_{\theta} - h^{u})}{R}\right),$$

where $\theta = \{\underline{\theta}, \overline{\theta}\}$. It depends positively on the return gap $(h^{e}G_{\overline{\theta}} - h^{u})$ and negatively on the return from saving R. Since talented borrowers have higher total and marginal returns in the successful state they decide to invest a higher level in education, i.e. $I_{\overline{\theta}}^{*} > I_{\underline{\theta}}^{*}$.

When agents spend more than the first-best amount on human capital, that is, when there is overinvestment, the expected marginal profits of the investment are below the riskless interest rate R. This means that agents are not investing properly. By merely reducing human capital investment and putting the excess bequest into the capital market at the riskless interest rate R, agents would increase their expected wealth. Accordingly, agents with an inherited wealth above I_{θ}^{*} will invest the first best amount and become lenders. When agents invest below the first best, and thus there is underinvestment, the expected marginal profits of the investment are above the riskless interest rate. It will be optimal for the agents to increase the investment in education until both rates of return are equal. Agents with an inherited wealth below the first best investment are therefore the ones who become borrowers.

In the next section we study the loan market for these applicants. We start by defining the contract and then we characterize the equilibrium.

2.2 The Financial Contract

Since banks know the bequest of each applicant, there is a submarket for each level of bequest. As they are unable to observe θ, banks cannot discriminate among borrowers at every b_t^j. Therefore, there is a continuum of contracts at each level of bequest. We will begin by analyzing the contract conditional on one specific level of bequest. After that we will see how this contract is modified when the inherited wealth changes.

Banks compete in two dimensions:

i) The rate of interest charged, F_{t+1} (one plus the interest rate on the loan).

ii) The amount of investment in education, I_t, so that the extent of the loan is determined by the investment in education minus the intergenerational transfer received, $(I_t - b_t^j)$.

Since the inherited wealth is observable the contract will be contingent on the borrowers' inherited wealth. Therefore, a bank's offer consists of a vector $\xi = (F_{t+1}(b_t), I_t(b_t))$ that specifies the interest rate, F_{t+1}, and the amount of investment in education, I_t, for any level of bequest.

Under asymmetric information, agents with an inherited wealth $b_t^j < I_{\underline{\theta}}^{*}$ become borrowers regardless of their talent and banks are unable to distinguish among borrowers of different abilities. Hence, banks offer the asymmetric information contract to these agents. As we have argued in the section above, low ability agents with $b_t^j \geq I^*_{\underline{\theta}}$ become lenders investing the first best amount $I^*_{\underline{\theta}}$. Similarly, high ability agents with $b_t^j \geq I^*_{\overline{\theta}}$ become lenders investing the first best amount $I^*_{\overline{\theta}}$. High ability agents with wealth $I^*_{\underline{\theta}} \leq b_t^j \leq I^*_{\overline{\theta}}$ do not have enough funds to invest the first best amount, and they thus apply to the capital market. Since only individuals of type $\overline{\theta}$ apply, the bank offers the full information contract to all of them. Therefore, the asymmetric information problem is only present for agents with $b_t^j < I^*_{\underline{\theta}}$.

Once agents invest in their education, the project could succeed or fail. In case of success, they become educated and earn an income high enough to repay their debt. In case of failure, agents become uneducated and earn an income so low that they cannot repay the debt. The borrowers' expected utility is thus given by their expected future wealth

$$U_{\theta,b_t^j} = p(\theta, I)[h^e G_\theta - F(I - b_t^j)] + (1 - p(\theta, I))h^u. \tag{3}$$

[Insert Figure 1].

Indifference curves $U_{\theta,b_t^j} = \overline{U}$ for the borrower are depicted in Figure1. The interest rate (F) is represented in the vertical axis and the investment in human capital (I) in the horizontal one. Each figure is drawn conditional on a certain level of bequest.[14] Since we are assuming marginal decreasing returns on the investment, the indifference curves are concave (see appendix B.1 for a proof of this property).

As the returns from the investment in education are higher for type $\overline{\theta}$, the marginal rate of substitution between investment and the interest rate is an increasing function of ability. Hence, high type borrowers are inclined to accept higher increases in the interest rate for a given increase in the amount of investment. Therefore, the indifference curves of a borrower satisfy the "single crossing" property. This fact enables banks to offer a pair of different contracts, where the loan size is used to reveal the ability of the borrower.

The utility increases in the southeast direction, where the quantity increases at a lower price. The dashed line $I^w_{\underline{\theta}}$ gives us the first best level of investment for the low type. As established in the previous section, $I^w_{\overline{\theta}}$ is situated at the right of $I^w_{\underline{\theta}}$.

We assume a competitive loan market and a small and open economy, so risk-neutral banks obtain their funds in a perfect capital market at the exogenous interest rate R. Because banks offer contracts with limited liability, the repayment is $F(I - b_t^j)$ in case of success and zero in case of failure. The banks' returns in expected terms are given by

$$\Pi_\theta = p(\theta, I)F(I - b_t^j) - R(I - b_t^j). \tag{4}$$

[14] We will see below that the higher the inherited wealth, the sharper the slope of the indifference curve for both types of agents.

Since the loan market is competitive, in equilibrium bank profits are zero. The break-even line ($F_\theta = \frac{R}{p(\theta,I)}$) of the bank in the plane (F,I) is downward sloping (see appendix B.2.). Contracts above the break-even line provide positive profits for the bank, while contracts below it provide losses. The zero iso-profit contour for high ability agents is below that of low ability agents (for each level of investment the interest rate is lower for them because they fail less often).

2.3 Characterization of the Equilibrium

We look for a pure strategy Nash Equilibrium in a two-stage game. In the first stage, each bank announces a pair of contracts $\{\xi_{\underline{\theta}}, \xi_{\overline{\theta}}\} = \{(F_{\underline{\theta}}, I_{\underline{\theta}}), (F_{\overline{\theta}}, I_{\overline{\theta}})\}$, for each level of bequest. In the second stage, borrowers simply select their most preferred loan contract from the set of all contracts offered by banks.

We allow for "free entry" so that an additional bank could always enter if a profitable contracting opportunity existed. For simplicity, we assume that a borrower can apply to only one bank during the period under consideration. Due to perfect competition, banks take others banks' offers as given.

Under these conditions, an equilibrium in a competitive market is a set of contracts such that:

i) Each contract $\{\xi_{\underline{\theta}}, \xi_{\overline{\theta}}\}$ guarantees nonnegative profits for the bank.

ii) Contracts announcements are incentive compatible in the presence of others announced contracts, that is, for any $b_t^j < I^*_{\underline{\theta}}$

$$p(\overline{\theta}, I_{\overline{\theta}})[(h^e G_{\overline{\theta}} - h^u) - F_{\overline{\theta}}(I_{\overline{\theta}} - b_t^j)] \geq p(\overline{\theta}, I_{\underline{\theta}})[(h^e G_{\overline{\theta}} - h^u) - F_{\underline{\theta}}(I_{\underline{\theta}} - b_t^j)] \ , \tag{5}$$

$$p(\underline{\theta}, I_{\underline{\theta}})[(h^e G_{\underline{\theta}} - h^u) - F_{\underline{\theta}}(I_{\underline{\theta}} - b_t^j)] \geq p(\underline{\theta}, I_{\overline{\theta}})[(h^e G_{\underline{\theta}} - h^u) - F_{\overline{\theta}}(I_{\overline{\theta}} - b_t^j)]. \tag{6}$$

iii) No bank has an incentive to offer an alternative set of profitable, incentive compatible contracts.

In part ii) we have introduced the incentive compatibility constraint as a restriction. Banks are unable to directly distinguish among borrowers. They can do so only by offering a pair $(\xi_{\underline{\theta}}, \xi_{\overline{\theta}})$ of different credit contracts that acts as a self-selection mechanism. These restrictions force borrowers to make choices in such a way that they reveal their types.

2.4 Discussion of Modeling Assumptions

The model assumes that parents obtain utility from bequests. This simplifies the dynamics of the model and allow us to obtain a closed-form solution. Assuming that parents were altruistic towards their children (i.e. they value the utility of their offspring) would substantially complicate the model. Under this assumption our model would need to be solved using a signaling framework.

In our model, banks compete in price and in quantities. Bose and Cothren (1997) and Becivenga and Smith (1993) use the interest rate, the amount of the loan, and the probability of rationing as instruments of the financial contract . In their models, lenders use credit rationing as a response to the adverse selection problem. The pivotal modeling difference between their analyses and ours is that in their models each borrower receives the same amount of investment. As a result, lenders can not discriminate with respect to the amount of investment or the interest rate (since there is perfect competition) and use credit rationing as the instrument to differentiate among agents. Note that the introduction of the probability of rationing as another instrument could easily be incorporated into our paper. In fact, our results do not change since everybody receives the loan and the distortion is still given by the amount of investment.

Besanko and Thakor (1987a; 1987b), and Bester (1985) use collateral as an instrument. The only role of collateral is to allow for self-selection of borrowers. In our model the loan size is variable, that is, I is divisible, and this helps us to separate borrowers at any level of inherited wealth. Therefore, when loans are of variable size, no collateral is required anymore.

3 The Equilibrium Contracts

In the next subsections we analyze the behavior of banks and borrowers. To provide a benchmark against which to measure the effects of information asymmetries, we first consider the equilibrium when there is full information.

3.1 Full Information

For any agent with an inherited wealth $b_t^j < I_\theta^*$ the bank solves the following problem: maximize a borrower's expected utility given by (3) subject to the participation constraint for the bank, equation (4), which holds with equality given the hypothesis of free entry and perfect competition among banks. It is straightforward to verify that for any high ability agent with wealth $b_t^j < I_{\overline{\theta}}^*$, and any less able applicant with $b_t^j < I_{\underline{\theta}}^*$, the equilibrium contract is given by $\xi^* = \{\xi_{\underline{\theta}}^*, \xi_{\overline{\theta}}^*\}$ with

$$\xi_\theta^* = (F_\theta^*, I_\theta^*) = \left(\frac{R}{p(\theta, I_\theta^*)}, \ln\left(\frac{B(\theta)(h^e G_\theta - h^u)}{R} \right) \right),$$

where $\theta = \{\underline{\theta}, \overline{\theta}\}$.[15] This equilibrium contract is depicted in Figure 1.

The interest rates charged to borrowers are entirely determined by the opportunity cost of funds and success probabilities. Therefore, the equilibrium contract $\xi^* = \{\xi_{\underline{\theta}}^*, \xi_{\overline{\theta}}^*\}$ is independent of the inherited wealth. The independence with respect to the inherited wealth is due to the risk neutrality of the agents. The implication of this result is that, independently of how wealth is distributed, people within each ability type will always invest their respective first best amount of resources in education.

[15] Notice that the only contract at which there is no profitable deviation is the Pareto optimal contract $\xi^* = \{\xi_{\underline{\theta}}^*, \xi_{\overline{\theta}}^*\}$.

High ability borrowers are better off than the low ability ones at any b_t, since they have higher returns when they succeed and they fail less often. Thus, it is not a surprise that under full information banks provide talented borrowers with better contracts (more funds at a lower interest rate).

Under $(A2)$ the contract $\xi^*_{\underline{\theta}}$ is not incentive compatible. That is, if ability is private information the contract $\{\xi^*_{\underline{\theta}}, \xi^*_{\overline{\theta}}\}$ is no longer an equilibrium since the low ability borrowers are strictly better off accepting the contract $\xi^*_{\overline{\theta}}$. Therefore, if a bank offers $\{\xi^*_{\underline{\theta}}, \xi^*_{\overline{\theta}}\}$ under private information it will obtain negative profits. The next section proposes the contract under asymmetric information.

3.2 Asymmetric Information

The equilibrium contract specifies the pair $(F_{t+1},\ I_t)$ offered to each θ type. The equilibrium could be a separating equilibrium, where different types choose different contracts, or a pooling equilibrium, where different types choose the same contract. Arguments identical to those given in Rothschild and Stiglitz (1976) establish that Nash equilibria are never pooling and any offer induce self-selection of borrowers.[16]

Under asymmetric information the equilibrium contract is characterized by the following proposition.

Proposition 1 *For any agent with $b^j_t < I^*_{\underline{\theta}}$, the equilibrium under asymmetric information (if it exists) is given by the credit contract $\xi' = \{\xi^*_{\underline{\theta}}, \xi^{'}_{\overline{\theta}}\}$ where*

$$\xi^*_{\underline{\theta}} = (F^*_{\underline{\theta}}, I^*_{\underline{\theta}}) = \left(\frac{R}{p(\underline{\theta}, I^*_{\underline{\theta}})}, \ln\left(\frac{B(\underline{\theta})(h^e G_{\underline{\theta}} - h^u)}{R} \right) \right),$$

and

$$\xi^{'j}_{\overline{\theta}} = (F^{'j}_{\overline{\theta}}, I^{'j}_{\overline{\theta}}) = \left(\frac{R}{p(\overline{\theta}, I^{'j}_{\overline{\theta}})}, I^{'j}_{\overline{\theta}} \right),$$

with $I^{'j}_{\overline{\theta}}$ given by

$$p(\underline{\theta}, I^*_{\underline{\theta}})[(h^e G_{\underline{\theta}} - h^u) - F^*_{\underline{\theta}}(I^*_{\underline{\theta}} - b^j_t)] = p(\underline{\theta}, I^{'j}_{\overline{\theta}})[(h^e G_{\overline{\theta}} - h^u) - F^{'j}_{\overline{\theta}}(I^{'j}_{\overline{\theta}} - b^j_t)]. \tag{7}$$

[Insert Figure 2].

[16] If pooling contracts are offered, then there exists another credit offer that is profitable because it attracts only high ability types from the pooling contract. Hence, a pooling contract is never viable against competition.

Moreover, with asymmetric information $IC_{\overline{\theta}} > 0$ (equation (5) is not binding), and high ability agents thus prefer the contract offered to the high type to the contract offered to the low type.[17]

Low type borrower receives the full information contract. The bank's incentive problem is to deter $\underline{\theta}$ type borrowers from choosing the contract of the $\overline{\theta}$ type borrowers. This incentive can be counteracted by making the $\overline{\theta}$ contract less favorable to $\underline{\theta}$ type borrowers, i.e. by "distorting" the first-best contract of the $\overline{\theta}$ type borrowers. This is the only way to ensure that the $\underline{\theta}$ type will satisfy the self-selection mechanism and will therefore not have incentives to choose the contract for the $\overline{\theta}$ type.

From Figure 2 (drawn conditional on a certain level of bequest) we can see that the bank distorts the high type borrowers by providing overinvestment, in that the amount of investment is higher than the first best. As a result, talented individuals are worse off under asymmetric information.

The intuition behind this overinvestment result is as follows: the interest rate is the instrument used to ensure the zero profit condition. Hence, the only way banks can sort out borrowers is by adjusting the investment level. Because the marginal rate of substitution between investment and the interest rate is an increasing function of ability, high ability borrowers are willing to pay more for an incremental amount of investment.[18] Therefore, investment can be used to reveal the borrowers' ability. Consequently, a contract specifying a suboptimal high investment is relatively more attractive for talented borrowers.

[Insert Figure 3].

If a high type borrower has stronger balance sheet positions, the distortion of the contract will be lower insofar as the amount of investment among high type ($I_{\overline{\theta}}^{\prime j}$) depends negatively on inherited wealth. The more a borrower invests in her own project, the less her interest will diverge from the interest of the bank. This greater compatibility of interest reduces the informational problem associated with the investment process. Thus, the distortion is lower when the inherited wealth increases, ceteris paribus. This can be seen by comparing Figures 2 and 3, where in Figure 3 the borrower has a higher inherited wealth than in Figure 2. The equilibrium amount of investment $I_{\overline{\theta}}^{\prime j}$ in Figure 3 (rich agent) is closer to $I_{\overline{\theta}}^{*}$ than it is $I_{\overline{\theta}}^{\prime j}$ (poor agent) in Figure 2. Note first that the level of utility is increasing with the inherited wealth, so that as b_t^j increases, the utility moves in a southeast direction. Second, from the Appendix we can check that the higher the inherited wealth, the sharper the slope of the indifference curve for both types of agents.

[17]In fact, one can easily check that $IC_{\overline{\theta}}$ is monotonically decreasing in b_t^j. Moreover, using equation (7) we can show that $IC_{\overline{\theta}}$ is positive when the bequest is equal to $I_{\underline{\theta}}^{*}$. Therefore $IC_{\overline{\theta}}(b_t^j) > 0 \; \forall b_t^j \in \left[\underline{b}, I_{\underline{\theta}}^{*}\right)$. See appendix B.5 for the proof.

[18]This makes sense since it is precisely the $\overline{\theta}$ type the one with a higher success probability and higher returns from the investment in her education.

In our model the riskiness of the venture, and thus the risk of default, is determined not just by ability but also by the level of investment. Ability affects the probability of failure. Moreover, under asymmetric information the amount of investment among high type depends negatively on the inherited wealth. Then, the interest rate charged to high ability types must decrease to reflect the change in default risk. In fact, banks react by lowering the interest rate charged to them ($F_{\overline{\theta}}^{\prime j} = \frac{R}{p(\overline{\theta}, I_{\overline{\theta}}^{\prime j})} < F_{\overline{\theta}}^{*}$). However, the debt repayment ($F_{\overline{\theta}}^{\prime j}(I_{\overline{\theta}}^{\prime j} - b_t^j)$) is higher among high ability types. As a result, the high types are worse off under asymmetric information even though mobility (as we will see in the next section) will be higher among them. In our model, the welfare cost of informational frictions are those associated with the presence of binding incentive constraints, and these costs are borne by talented agents.

Clearly, since $\underline{\theta}$ type borrowers receive the first best contract they always prefer to borrow than to become self-financed. Because talented borrowers are the ones who face distortions, they may prefer to refuse the contract and invest only their inherited wealth in their education. However, in equilibrium these agents choose to borrow rather than to fully self-finance their investment. In fact, one can show that the higher the bequest received by an individual, the more incentives she will have to refuse the loan.[19] Therefore it is enough to show that the incentive compatibility constraint is not binding for a high type when $b_t^j = I_{\underline{\theta}}^*$, which has been proven above.

Once we have characterized the candidate separating equilibrium we need to be completely sure that there is no way to distort our proposed equilibrium. That is, we need to check that no banks have an incentive to offer an alternative set of profitable, incentive compatible contracts. By construction, no bank has an incentive to offer any other contract which attracts only one type of borrower. Thus, there is no loan contract that low ability borrowers prefer to $\xi_{\underline{\theta}}^*$ which earns non-negative profits when only low types accept it. Similarly, there is no incentive-compatible loan contract that high type prefers to $\xi_{\overline{\theta}}^*$ which

[19] In fact it is possible to show that

$$\frac{\partial U_{\overline{\theta},\underline{\theta}}}{\partial b_t} < \frac{\partial U_{\overline{\theta}}^A}{\partial b_t}, \tag{8}$$

where $U_{\overline{\theta},\underline{\theta}}(b_t)$ represents the utility of a talented agent when she chooses the low type contract, while $U_{\overline{\theta}}^A(b_t)$ is her utility when she does not go to the capital market.

Moreover, we have previously proven that

$$\frac{\partial U_{\overline{\theta},\overline{\theta}}}{\partial b_t} < \frac{\partial U_{\overline{\theta},\underline{\theta}}}{\partial b_t}, \tag{9}$$

therefore by (8) and (9) we get the following result

$$\frac{\partial \left(U_{\overline{\theta},\overline{\theta}} - U_{\overline{\theta}}^A \right)}{\partial b_t} < 0. \tag{10}$$

See Appendix B.5 for the proof.

earns non-negative profits when it is taken by high ability individuals only. As a consequence, an equilibrium exists if and only if no bank has an incentive to offer a pooling contract. Since Nash equilibria are never pooling we need to check under which conditions pooling contracts are never offered. If we find these conditions, our equilibrium exists and it is the one characterized by Proposition 1. In a pooling contract the losses that banks suffer with the contract offered to $\underline{\theta}$ types are offset by the profits of the $\overline{\theta}$ type contract. Therefore, when the probability of being a low type is very small, the incentives to have a pooling contract increase. Hence, proposition 2 tells us that in order to have the separating equilibrium, the proportion of low ability agents needs to be high enough.

First, assume that the exogenous parameters take the form

$$\frac{B(\overline{\theta})}{B(\underline{\theta})}[1 - \frac{h^e G_{\underline{\theta}} - h^u}{h^e G_{\overline{\theta}} - h^u}] < \ln(\frac{h^e G_{\overline{\theta}} - h^u}{h^e G_{\underline{\theta}} - h^u}). \tag{A3}$$

Then, under $A3$ the following proposition holds

Proposition 2 *Let $(\widetilde{F}, \widetilde{I})$ be the pooling contract offered by the bank, $V_{\overline{\theta}}^{P}(.)$ the indirect utility function of a talented borrower applying for the pooling contract, and $V_{\overline{\theta}}^{S}(.)$ the indirect utility when he applies for the separating contract. Then, if $\gamma > \widetilde{\gamma}(b_t = \underline{b})$ the following inequality holds:*

$$V_{\overline{\theta}}^{P}(\widetilde{F}(\gamma), \widetilde{I}(\gamma), p(\overline{\theta}, \widetilde{I}(\gamma)), b_t^j) < V_{\overline{\theta}}^{S}(F_{\overline{\theta}}^{\prime j}, I_{\overline{\theta}}^{\prime j}(b_t^j), p(\overline{\theta}, I_{\overline{\theta}}^{\prime j}), b_t^j), \tag{11}$$

for $b_t^j \in [\underline{b}, I_{\underline{\theta}}^*)$ with $\underline{b}$ being the lowest possible level of inherited wealth, and the equilibrium is the separating one.

Notice that this proposition extends the result found by Rothschild and Stiglitz (1976). In their model all agents have the same amount of initial wealth. They found that when the proportion of low ability borrowers is higher than a certain threshold level, the separating equilibrium exists. By contrast, in our model agents differ also in the inherited wealth which is endogenously provided. Consequently, our threshold level depends on the inherited wealth, $\widetilde{\gamma}(b_t)$, and more specifically, it decreases with inherited wealth. Therefore, if we guarantee the existence of equilibrium for the lowest level of bequest (as we will see later, this is given by $\underline{b} = \alpha h^u$) we have equilibrium for higher levels.

3.3 Discussion

Our result that there is overinvestment is in contrast to the conventional underinvestment outcome implicit in the microeconomic literature that analyses adverse selection between banks and borrowers.

It is worthwhile considering the differences in terms of assumptions and results between the work by Stiglitz and Weiss (1981) and our paper. In their paper credit rationing appears because the expected return received by the bank decreases at some point with the rate of interest charged to borrowers. This is due to an adverse selection effect which appears when a rise in the interest rate changes the mix of applicants in an adverse way in that safe potential borrowers drop out of the market, lowering the average quality of borrowers. In our model, by contrast, an increase in interest rates will decrease (instead of increase) the average risk (or similarly increase the average ability) of the population of borrowers. Thus, in our model the marginal project financed (that for which the borrower is indifferent between applying to the capital market or becoming self financed) has the lowest success probability, whereas in the Stiglitz and Weiss model it has the highest. Therefore, in Stiglitz and Weiss' model the bank may prefer to reject some borrowers instead of increasing the interest rate. They obtain pure credit rationing since some individuals obtain loans whereas apparently identical individuals who are willing to borrow on precisely the same terms do not.

One crucial assumption behind the underinvestment result is that in Stiglitz and Weiss all borrowers have the same expected profits but the dispersion of the profits is different, whereas in our model the expected profits differ among borrowers (in fact, talented borrowers have higher expected profits than less able ones at any b_t^j). Another important assumption for having credit rationing is that in their model debt contracts are imposed exogenously and the contract therefore does not allow for any sorting mechanism to be constructed in such a way that each type of borrower will choose a specific type of contract. By contrast, in our model self-selection of borrowers will result from product differentiation since the loan size differs among agents, and thus could be used to separate out agents. And if separation is complete, rationing can no longer occur.

Our overinvestment result depends on a number of assumptions, though the key appears to be the complementarity between ability and the investment as well as the assumption that ability positively affects the returns from the investment in education. There are some papers where borrowers overinvest in equilibrium. Besanko and Thakor (1987a) find that lower risk borrowers get more credit in equilibrium than they would with full information. The basic assumption generating this result is that the marginal rate for substitution between investment and interest rate is an increasing function of the success probability. Our fundamental assumptions imply exactly the same. De Meza and Webb (1987) also find that borrowers invest in excess of the socially efficient level. They assume that banks cannot determine whether an individual consumer holds loans from other banks, and as a consequence the equilibrium will be a pooling equilibrium rather than a separating one.[20] Overinvestment is

[20] Rothschild and Stiglitz's (1976) proof that there cannot be a pooling equilibrium depends on the assumption that borrowers can buy only one contract, an assumption which also holds in our model. The implication of this assumption is, in effect, that the bank specifies both the prices and quantities in the contract. There exists, therefore, price and quantity competition among banks. As Rothschild and Stiglitz point out, the assumption that borrowers can buy only one contract is an objectionable one. By arguing that there is absence of monitoring purchases from banks, De Meza and Webb use a price competition framework and borrowers

obtained because in their model an increase in the interest rate would decrease the average risk of the population of borrowers, and credit rationing would consequently never occur at the equilibrium interest rate.

4 Mobility, Inequality and Education

4.1 Full Information

The distribution of bequests in period t is given by $G_t(b)$. As we have normalized the mass of population to one, $G_t(b)$ also represents the fraction of the population with current wealth below b. We will show that the distribution of wealth converges to a unique steady state distribution, independently of the initial conditions, so that historical endowments do not matter in the long run. In order to do that, we need to define the way in which bequests evolve.

The agents' optimal decisions (see subsection 3.1) and the stochastic process of the shocks (ability and investment shocks) determine the Markov process of bequests. With full information the bequest follows a linear Markov process of the form $b_{t+1}^j = \alpha y_{t+1}^j$, where the realized income is given by the equations written below. The investment in education can be successful (an event that occurs with probability $\overline{p} = p(\overline{\theta}, I_{\overline{\theta}}^*)$ if agents are of high ability), or can be unsuccessful (an event that occurs with probability $1-\overline{p} = 1-p(\overline{\theta}, I_{\overline{\theta}}^*)$ if agents are of high ability). The law of motion of bequest is given by the equations below.

With probability $(1-\gamma)\overline{p}$ we deal with successful high ability agents, so the bequests evolve as:

$$b_{t+1}^j = g(b_t^j, \overline{\theta}, \overline{p}) = \begin{cases} \alpha[h^e G_{\overline{\theta}} + R(b_t^j - I_{\overline{\theta}}^*)] & if \quad b_t^j \geq I_{\overline{\theta}}^* \\ \alpha[h^e G_{\overline{\theta}} - F_{\overline{\theta}}^*(I_{\overline{\theta}}^* - b_t^j)] & if \quad b_t^j < I_{\overline{\theta}}^*. \end{cases} \tag{12}$$

where the third argument in the function $g()$ indicates that the agent has succeeded. With probability $(1-\gamma)(1-\overline{p})$ we deal with defaulting high ability agents, so the bequests evolve as:

$$b_{t+1}^j = g(b_t^j, \overline{\theta}, 1-\overline{p}) = \begin{cases} \alpha[h^u + R(b_t^j - I_{\overline{\theta}}^*)] & if \quad b_t^j \geq I_{\overline{\theta}}^* \\ \alpha h^u & if \quad b_t^j < I_{\overline{\theta}}^*. \end{cases} \tag{13}$$

where the third argument in the function $g()$ indicates that the agent has not succeeded. With probability $\gamma\underline{p}$ we deal with successful low ability agents, so the bequests evolve as:

$$b_{t+1}^j = g(b_t^j, \underline{\theta}, \underline{p}) = \begin{cases} \alpha[h^e G_{\underline{\theta}} + R(b_t^j - I_{\underline{\theta}}^*)] & if \quad b_t^j \geq I_{\underline{\theta}}^* \\ \alpha[h^e G_{\underline{\theta}} - F_{\underline{\theta}}^*(I_{\underline{\theta}}^* - b_t^j)] & if \quad b_t^j < I_{\underline{\theta}}^*, \end{cases} \tag{14}$$

are thus allowed to buy arbitrary multiples of contracts offered.

and finally, with probability $\gamma(1-\underline{p})$ we deal with defaulting low ability agents, thus the bequests evolve as:

$$b_{t+1}^j = g(b_t^j, \underline{\theta}, 1-\underline{p}) = \begin{cases} \alpha[h^u + R(b_t^j - I_{\underline{\theta}}^*)] & if \quad b_t^j \geq I_{\underline{\theta}}^* \\ \alpha h^u & if \quad b_t^j < I_{\underline{\theta}}^*. \end{cases} \tag{15}$$

Note that the functions g are time independent.

[Insert Figure 4].

The graph of the law of motion of the bequest with full information is given in Figure 4. Here we draw the transition function for a high type agent (equations 12 and 13) and a low type agent (equations 14 and 15). On the horizontal axis we have the inherited wealth, b_t, and on the vertical axis the bequest given to the child, b_{t+1}.

Define the highest possible wealth of an uneducated agent as $x = \alpha[h^u + R(\overline{b} - I_{\underline{\theta}}^*)]$. This is the second period income of a low ability agent who invests the optimal amount $I_{\underline{\theta}}^*$ in education and fails. In particular, this individual receives the highest possible bequest, $\overline{b}$, and she invests the excess of capital $(\overline{b} - I_{\underline{\theta}}^*)$ in the capital market. Similarly, let us define the lowest possible wealth of an educated agent as $z = \alpha[h^e G_{\underline{\theta}} - F_{\underline{\theta}}^*(I_{\underline{\theta}}^* - \underline{b})]$. Note that both x and z do not change under asymmetric information. We assume that x is smaller than z. In turn, we restrict our attention to parameter values such that one-period catch-up in terms of income between very poor (i.e., agents endowed with a very low bequest) but educated agents and very rich but uneducated agents is possible. This assumption will be very useful when we endogenously compute the probability of upward and downward mobility.

The highest sustainable wealth for a high type is $\overline{b} = \frac{\alpha}{1-\alpha R}[h^e G_{\overline{\theta}} - R I_{\overline{\theta}}^*]$.[21] The lowest sustainable wealth is given by $\underline{b} = \alpha h^u$. We assume that

$$\alpha R < 1. \tag{A4}$$

Because $A4$ holds, if the inherited wealth is smaller than or equal to $\overline{b}$ it can never exceed $\overline{b}$ at any time. Likewise, if the inherited wealth is greater or equal to $\overline{b}$ the wealth of the dynasty will become less than or equal to $\overline{b}$. Therefore, we restrict our analysis to the interval $\beta = [\underline{b}, \overline{b}]$ and define the support of the distribution of bequest in this interval.

Given that there is a continuum of agents and that both ability and the returns from the investment are *i.i.d.* random variables, the distribution function of the aggregate wealth can be interpreted as a deterministic variable by the law of large numbers. The bequest distribution G_{t+1} in period $t+1$ is obtained from the distribution in period t by adding up the total mass of agents who end up with a bequest less than b_{t+1}. Therefore, the bequest distribution $G_{t+1}(b)$

[21] Note that in order to have $\overline{b} > I_{\overline{\theta}}^*$,we need $\alpha h^e G_{\overline{\theta}} > I_{\overline{\theta}}^*$.

evolves over time as dictated by the following functional equation:

$$\begin{aligned} G_{t+1}(b) &= \gamma[(1-\underline{p}) \int_{\underline{b}}^{\phi(b,\underline{\theta},0)} dG_t(b) + \underline{p} \int_{\underline{b}}^{\phi(b,\underline{\theta},1)} dG_t(b)] \\ &\quad +(1-\gamma)[(1-\overline{p}) \int_{\underline{b}}^{\phi(b,\overline{\theta},0)} dG_t(b) + \overline{p} \int_{\underline{b}}^{\phi(b,\overline{\theta},1)} dG_t(b)], \end{aligned} \tag{16}$$

where $\phi(b,\overline{\theta},1) = g^{-1}(b_{t+1},\overline{\theta},1)$. More precisely, $\phi(b,\theta,1) = \{b \geq 0$ such that $g(b_t,\theta,\delta) \leq b\}$.

We can prove the existence, uniqueness and convergence of the invariant distribution with full information by using Hopenhayn-Prescott's (1992) analysis. Picture 4 gives an intuition of this result. In our model the fact that everybody has access to the capital market as well as the fact that everybody may fail with positive probability allows the individuals within a dynasty to move along the different values of the wealth distribution. When dynasty wealth may move from any measurable subset $[\underline{b},\overline{b}]$ to any other measurable subset of $[\underline{b},\overline{b}]$ the Markov process has a unique invariant distribution.

Proposition 3 *There exists a unique invariant distribution G^{FI} for the Markov process corresponding to the transition function $P(b,A)$. Irrespective of the initial wealth distribution G_0^{FI}, the sequence $(T^*)^n G_0^{FI}$ converge to G^{FI}, where T^* is the operator defined by (16).*

Since shocks on individual investments are idiosyncratic, there will be some inequality in the long-run, but this inequality is independent of the initial inequality $G_0(b)$. Thus, even though wealth inequality cannot be completely eliminated, in the long run all dynasties fare equally well on average.

We can easily compute the number of educated and uneducated people. Anybody with second period wealth below x is uneducated. The number of educated is agents is

$$1 - G^{FI}(x) = (1-\gamma)\overline{p} + \gamma\underline{p} = 1 - p(U_{t+1}) = p(E_{t+1}). \tag{17}$$

We now define intergenerational mobility among the educated or/and uneducated. Intergenerational mobility is measured by computing the transition matrix between these two classes, say $p(j/i)$, $i = e,u; j = e,u$, where $p(U_{t+1}/E_t)$ is the probability that an individual, whose parent was educated, becomes uneducated. This is the probability of having downward mobility. Under full information this probability is

$$p(U_{t+1}/E_t) = \frac{[(1-\gamma)(1-\overline{p}) + \gamma(1-\underline{p})][1-G^{FI}(x)]}{1-G^{FI}(x)}.$$

Similarly, upward mobility is the probability that children of uneducated parents become educated. This probability is given by

$$p(E_{t+1}/U_t) = (1-\gamma)\overline{p} + \gamma\underline{p}.$$

In our model these transitions probabilities are endogenous. If capital markets function perfectly, individuals invest in education until the expected rate of return equals the rate of return on physical capital no matter what their family background is. In other words, independently of how wealth is distributed, poor and rich people with the same ability will invest the same amount. Therefore, there is no correlation between inherited wealth and education, and thus the events U_t and E_{t+1} are stochastically independent. Hence, the inherited wealth does not affect the probability of becoming educated $p(E_{t+1})$, i.e. $p(E_{t+1}/U_t) = p(E_{t+1}/E_t) = p(E_{t+1})$, and likewise, $p(U_{t+1}/U_t) = p(U_{t+1}/E_t) = p(U_{t+1})$. In our model the only variable linking the periods is the inherited wealth. Because the full information contract is given by the first best, which is independent of inherited wealth, we do not have any dynamics under full information, and thus $p(E_t) = p(E_{t+1})$ at any t.

4.2 Asymmetric Information

With asymmetric information the distribution of wealth matters for analyzing mobility. Thus, aggregate statistics (output and aggregate human capital) do not depend only on the types of agents and the investment cost in education, but also on the financial situation of the agents (captured here by the distribution of the initial wealth). What is important now is how wealth is distributed among agents. It is worth remembering that the low types receive the first best contract, so the evolution of their bequests is given by equations (12) and (13). In case of success, this law of motion can be written as follows,

$$g(b_t^j, \overline{\theta}, \cdot) = \begin{cases} \alpha[h^e G_{\overline{\theta}} + R(b_t^j - I^*_{\overline{\theta}})] & if \quad b_t^j \geq I^*_{\overline{\theta}} \quad with \quad (1-\gamma)\overline{p} \\ \alpha[h^e G_{\overline{\theta}} - F^*_{\overline{\theta}}(I^*_{\overline{\theta}} - b_t^j)] & if \quad I^*_{\underline{\theta}} \leq b_t^j < I^*_{\overline{\theta}} \quad with \quad (1-\gamma)\overline{p} \\ \alpha[h^e G_{\overline{\theta}} - F'_{\overline{\theta}}(I'_{\overline{\theta}} - b_t^j)] & if \quad b_t^j < I^*_{\underline{\theta}} \quad with \quad (1-\gamma)p(\overline{\theta}, I'_{\overline{\theta}}). \end{cases} \tag{18}$$

Instead, when educational investment is not successful, bequests evolve according to

$$g(b_t^j, \overline{\theta}, \cdot) = \begin{cases} \alpha[h^u + R(b_t^j - I^*_{\overline{\theta}})] & if \quad b_t^j \geq I^*_{\overline{\theta}} \quad with \quad (1-\gamma)(1-\overline{p}) \\ \alpha h^u & if \quad b_t^j < I^*_{\overline{\theta}} \quad with \quad (1-\gamma)(1-p(\overline{\theta}, I'_{\overline{\theta}})). \end{cases} \tag{19}$$

Note that, when credit markets are imperfect, the equality between the marginal product of human capital and the interest rate does not hold. As we will see below, the correlation between inherited wealth and ability will in fact have an important effect on intergenerational mobility. Because of this correlation, (18) and (19) for the high type and equations (12) and (13) for the low type define a non-linear aggregate transition function $G_{t+1}(G_t)$.

The graph of the law of motion of the bequests with asymmetric information is very similar to the one under full information.[22] The only thing that changes is the bequest function for high ability agents with wealth $b_t^j < I_{\underline{\theta}}^*$. This new bequest function is below the full information one. Since the distortion is higher at low levels of inherited wealth, the gap between the bequest function with full and asymmetric information is higher in this range. The bequest function is upward sloping but steeper with asymmetric information.

The aggregate bequests distribution satisfies

$$\begin{aligned} G_{t+1}(b) &= \gamma[(1-\underline{p}) \int_{\underline{b}}^{\phi(b,\underline{\theta},0)} dG_t(b) + \underline{p} \int_{\underline{b}}^{\phi(b,\underline{\theta},1)} dG_t(b)] \\ &\quad +(1-\gamma)[\int_{\underline{b}}^{\phi(b,\overline{\theta},0)} (1-\overline{p}')dG_t(b) + \int_{\underline{b}}^{\phi(b,\overline{\theta},1)} \overline{p}' dG_t(b)], \end{aligned} \tag{20}$$

with $\phi(b,\theta,\cdot) = \{b \geq 0 \text{ such that } g(b_t,\theta,0) \leq b\}$ and $\overline{p}' = p(\overline{\theta}, I'_{\overline{\theta}}(b_t))$.

Using the same argument as with full information, we can prove the existence of an invariant distribution of bequests, G^{AI}, where AI denotes asymmetric information.

Proposition 4 *There exists a unique invariant distribution G^{AI} for the Markov process corresponding to $P(b,A)$. For any given G_0^{AI}, the sequence $(T^*)^n G_0^{AI}$ converge to G^{AI}, where T^* is the operator defined by (20).*

With asymmetric information the number of educated individuals can be computed using (20). Remember that talented agents with an inherited wealth $b_t^j < I_{\underline{\theta}}^*$ will now become borrowers and their success probability will be different than in the full information case. Consequently, the events U_t and E_{t+1} are not stochastically independent and thus $p(E_{t+1}/U_t) \neq p(E_{t+1})$.

The number of educated agents is

$$1 - G^{AI}(x) = p(E_{t+1}) = (1-\gamma)[\int_{\underline{b}}^{I_{\underline{\theta}}^*} p(\overline{\theta}, I'_{\overline{\theta}}(b_t))dG^{AI}(b) + \overline{p}(1-G^{AI}(I_{\underline{\theta}}^*))]. + \gamma\underline{p}. \tag{21}$$

[22] Remember that a $\underline{\theta}$ type receives the full information contract and that rich agents invest the first best amount. Hence, the law of motion of the bequest does not change either for rich agents (regardless of their type) or for $\underline{\theta}$ type borrowers (regardless of their wealth).

We can compute the number of uneducated agents as,

$$G^{AI}(x) = (1-\gamma)[\int_{\underline{b}}^{I^*_{\underline{\theta}}} (1 - p(\overline{\theta}, I'_{\overline{\theta}}(b_t)))dG^{AI}(b) + (1-\overline{p})(1 - G^{AI}(I^*_{\underline{\theta}}))] + \gamma(1-\underline{p}).$$

Comparing the probabilities of becoming educated under full and asymmetric information, we conclude that since low ability agents receive the full information contract and talented borrowers are "distorted" in equilibrium, the probability of becoming educated among high ability borrowers is higher. Consequently, the level of human capital in equilibrium is higher under asymmetric information.

Proposition 5 *In the steady state the number of educated agents is higher with asymmetric information than with full information.*

In terms of educational outcomes, in a more mobile society the probability of being educated is higher than in a less mobile one. In our paper imperfections in the capital market cause a distortion among talented agents since they invest in education in excess the socially efficient level. This overinvestment enhances the probability of success and causes higher upward mobility than with full information. More specifically, the next proposition tells us that the asymmetry of information promotes upward mobility among talented borrowers. Note that a formal proof of this results is not a simple task because the distribution of wealth is endogenous and thus differs according to whether there is full or asymmetric information in an economy.

Proposition 6 *In the steady state upward mobility is higher with asymmetric information than with full information.*

Finally, note that this model is not suited to study the connections between intergenerational mobility and cross-sectional inequality, since we find opposing effects making inequality very difficult to evaluate. In particular, poor and talented agents are affected by the asymmetry of information in two opposite ways. On the one hand, since the success probability is higher, there are more people being educated. But, on the other, these talented borrowers have a lower wealth.

4.3 Empirical Evidence on Credit Constraints

What empirical evidence do we have about the extent to which credit constraints contribute to making inequality more persistent across generations? In order to answer this question we need evidence that gives a precise estimate of the extent to which credit constraints are likely to affect aggregate intergenerational mobility at the macro level.

Unfortunately, the empirical evidence that is currently available on the importance of credit constraints in intergenerational mobility is sparse. Moreover, the majority of the studies about credit constraints analyze whether borrowing constraints affect educational attainment. Clearly, if borrowing constraints are binding, then youths from families with less financial resources (those with less educated parents) will face a higher implicit schooling costs. The empirical evidence is very contradictory. The typical view (see Kane (1994) and Ellwood and Kane (2000)) stresses the importance of credit constraints for educational attainment. The positive correlation between family income and schooling has been widely interpreted as evidence supporting the idea that borrowing constraints hinder educational choices.

There are, however, a number of potential problems with this empirical work.[23] Recent studies by Cameron and Heckman (1998), Cameron and Taber (2001), Keane and Wolpin (2001) and Shea (2002) have attempted to shed more light on the determinants of schooling choices. Using very different methods, these researchers have found little evidence that favors the idea that borrowing constraints hinder college-going or any other schooling choice. For example, James J. Heckman (2002) examines arguments about the strength of credit constraints in schooling that are made in the literature, evaluating the available evidence and presenting new facts using American data. Heckman studies the relationship between family income and college enrollment and the evidence on credit constraints in post-secondary schooling. He draws a distinction between short-run liquidity constraints, which affect the resources required to finance college education, and the long-term factors that promote cognitive and noncognitive ability.[24] The latter emphasizes the long-run factors associated with higher family income and is consistent with another type of credit constraint: the inability of the child to buy the parental environment and genes that form the cognitive and noncognitive abilities required for success in school. His conclusion is that long run factors are more important, even though he identifies a group of people (at most 8% of the population) who seem to be facing short-run credit constraints.

It might be argued that borrowing constraints cannot have any important influence on college attendance decisions, given the existing programs available. However, the maximum Pell grant has generally been well below half of most estimates of tuition, room, and board cost (see Kane, 1994). Further, an individual's grant cannot exceed a certain fraction of college expenses (set at 50% during most of the time). Thus, the large range of subsidies to education alone do not cover the entire cost of a college education. Therefore, we may conclude that given the range of subsidies to education that only partially subsidize college expenses, there is no evidence of barriers to investment in education related to borrowing constraints.

Contrary to the "classical" view our model suggests that, once endogenized, credit constraints do not represent a barrier to investment in education, and

[23] See Heckman (2002) for an evaluation of the available evidence.

[24] Examples of noncognitive abilities are motivation, tenacity, trustworthiness and perseverance, among others.

thus to intergenerational mobility. Our conclusions, therefore, are consistent with the new assessment of the limited role of short-run credit constraints. However, further empirical work and the use of richer panel data sets is needed to shed light on these issues.

5 Conclusions

There is a conventional view (see authors like Becker and Tomes (1981), Loury (1981), Galor and Zeira (1993)) that CMI represent a barrier to intergenerational mobility. However, this branch of the literature assumes that CMI are exogenous and thus there is no microfoundation about the imperfections in the capital market.

In contrast to these analyses this paper tries to address the following questions: To what extent and through what mechanism do asymmetries of information between borrowers and banks affect inequality and intergenerational mobility? Is the nature of CMI important for understanding inequality and mobility? That is, are these previous results sensitive to the way we model imperfections in the capital market? What we do in this paper is study inequality and mobility in a model where CMI are endogenous. To this end we construct a growth model with adverse selection problems in the financial sector.

The major results of this paper might be summarized as follows: endogenous CMI may promote intergenerational mobility among talented individuals, since talented children from poor families get educated even more that they wish, so that both income mobility and human capital accumulation are larger than in the first best world. In this way, low ability individuals do not prefer the contract offered to the high ability ones.

Our main conclusion is that the nature of CMI is crucial to understanding its effects on intergenerational mobility.

6 References

AGHION, P. and BOLTON, P. (1997), "A Trickle-Down Theory of Growth and Development," *Review of Economic Studies*, 64, 151-172.

AGHION, P. and BOLTON, P. (1992), "Distribution and Growth in Models of Imperfect Capital Markets," *European Economic Review,* 36, 603-611.

AZARIADIS C. and SMITH, B.D. (1993), "Adverse Selection in the Overlapping Generations Model: The Case of Pure Exchange," *Journal of Economic Theory*, 60, 277-305.

BANERJEE, A. and NEWMAN, A. (1991), "Risk-Bearing and the Theory of Income Distribution," *Review of Economic Studies*, 58, 211-235.

BANERJEE, A. and NEWMAN, A. (1993), "Occupational Choice and the Process of Development," *Journal of Political Economy*, 101, 274-299.

BECKER, GARY S. and TOMES, N. (1986), "Human Capital and the Rise and Fall of Families," *Journal of Labor Economics*, vol. 4, n 3, 1-39.

BENCIVENGA, V.R. and SMITH, B.D. (1993), "Some Consequences of Credit Rationing in an Endogenous Growth Model," *Journal of Economic Dynamics and Control*, 17, 97-122.

BESANKO, D. and THAKOR, A.V. (1987a), "Competitive Equilibrium in the Credit Market under Asymmetric Information," *Journal of Economic Theory*, 42, 167-182.

BESANKO, D. and THAKOR, A.V. (1987b), "Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets," *International Economic Review*, 28 (3), 671-689.

BESTER, H. (1985), "Screening vs. Rationing in Credit Markets with Imperfect Information," *American Economic Review*, 75 (4), 850-855.

BOSE, N. and COTHREN, R. (1997), "Asymmetric Information and Loan Contracts in a Neoclassical Growth Model," *Journal of Money, Credit and Banking*, 29 (4), 423-39.

CAMERON, S. and HECKMAN, J. (1998), "Life Cycle Schooling and Dynamic Selection Bias: Models and Evidence for Five Cohorts of American Males," *Journal of Political Economy*, 106 (2), 262-333.

CAMERON, S. and TABER, C. (2001), "Borrowing constraints and the Returns to Schooling," NBER, Working Paper n7761.

CARNEIRO, P. and J. HECKMAN, J. (2002), "The Evidence on Credit Constraints in Post-secondary Schooling," *Economic Journal*, 112 (482), 705-734.

CHAMBERS, D. L. (1994), "The Burdens of Educational Loans: The Impact of Debt on Job Choice and Standards of Living for Students at Nine American Law Schools," *Journal of Legal Education*, 42,187-231.

DE MEZA, D. and WEBB D.C. (1987), "Too Much Investment: A Problem of Asymmetric Information," *Quarterly Journal of Economics*, 102, 281-292.

ELLWOOD, D. and KANE, T. (2000), "Who is Getting a College Education?: Family Background and the Growing Gaps in Enrollment," in S. Danzinger and K. Waldfogel, eds., *Securing the Future*. New York: Russel Sage.

GALOR, O. and MOAV, O. (1998), "Ability Biased Technological Transition, Wage Inequality and Growth," *Quarterly Journal of Economics*, 115(2), 469-498.

GALOR, O. and TSIDDON, (1997), "Technological Progress, Mobility, and Growth," *American Economic Review*, 87, 363-382.

GALOR, O. and ZEIRA J. (1993), "Income Distribution and Macroeconomics," *Review of Economic Studies*, 60, 35-52.

HOPENHAYN, H. and PRESCOTT, E. (1992), "Stochastic Monotonicity and Stationary Distributions for Dynamic Economies," *Econometrica,* 60, 1389-10405.

JAFFEE, D. and RUSSELL, T. (1976), "Imperfect Information, Uncertainty, and Credit Rationing," *Quarterly Journal of Economics*, 90, 651-666.

KANE, T. (1994), "College Entry by Blacks since 1970: The Role of College Costs, Family Background, and the Returns to Education, " *Journal of Political Economy*, 102(5): 878-911.

KEANE, M. and WOLPIN, K. (2001), "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment," *International Economic Review*, 42(4): 1051-1103.

LOURY, G. C. (1981), "Intergenerational Transfers and the Distribution of Earnings," *Econometrica*, 49, 843-867.

MANSKI, C. and WISE, D. (1983), "College Choice in America," University Chicago Press.

MAOZ, Y.D. and MOAV, O. (1999), "Intergenerational Mobility and the Process of Development," *Economic Journal,* 109, 677-97.

MILDE, H. and RILEY, J.G. (1988), "Signaling in Credit Markets," *Quarterly Journal of Economics*, 103 (1),101-129.

MULLIGAN, C.D. (1992), "*Parental Priorities and Economic Inequality,*" The University of Chicago Press.

OWEN, A.L. and WEILL, D. N. (1998), "Intergenerational Earnings Mobility, Inequality and Education," *Journal of Monetary Economics,* 41(1), 71-104.

PIKETTY, T. (1997), "The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing," *Review of Economic Studies,* 64, 173-189.

ROTHSCHILD, M. and STIGLITZ, J. (1976), "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90, 628-49.

SHEA, J. (2002), "Does Parent's Money Matter?," *Journal of Public Economic*, 77 (2), 155-184.

STIGLITZ, J.E. and WEISS, A. (1981), "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71, 393-410.

STINEBRICKNER, T. and STINEBRICNER, R. (2001), "The Underlying Relationship between Family Income and Schooling Attainment: Evidence form a Liberal Arts College with a Full Tuition Subsidy Program," Manuscript. University of Western Ontario.

STOKEY, N., R. LUCAS, and PRESCOTT, E. (1989), *Recursive Methods in Economic Dynamic,* Cambridge, MA: Harvard University Press.

7 *Appendix*

Appendix A: Proof of propositions.

Proof of Proposition 1.

First, we have to prove that in any separating equilibrium the low ability borrower receives the full information efficient contract, $\xi^*_{\underline{\theta}} = (F^*_{\underline{\theta}}, I^*_{\underline{\theta}})$. This contract must be on the zero profit line, and thus $F^*_{\underline{\theta}} = \frac{R}{p(\underline{\theta},I)}$. Any $I_{\underline{\theta}} \neq I^*_{\underline{\theta}}$ is strictly worse for the agent, and then it would be possible for the bank to Pareto improve it. Therefore, the only equilibrium contract for the low type is $\xi^*_{\underline{\theta}}$.

Secondly, in any separating equilibrium, high ability borrowers accept the contract $\xi^{\acute{j}}_{\overline{\theta}} = (F^{\acute{j}}_{\overline{\theta}}, I^{\acute{j}}_{\overline{\theta}})$ where $I^{\acute{j}}_{\overline{\theta}}$ satisfies the incentive compatibility constraint for a low type with equality.

The contract $\xi^{\prime j}_{\overline{\theta}}$ is found at the intersection of the $\underline{\theta}$ indifference curve that passes through $\xi^*_{\underline{\theta}}$ and the line $F^{\prime j}_{\overline{\theta}} = \frac{R}{p(\underline{\theta}, I^{\cdot})}$. There is also no contract that could make the borrower of type $\overline{\theta}$ better off than $\xi^{\prime j}_{\overline{\theta}}$ without either rendering losses to the bank or attracting the borrower of type $\underline{\theta}$ from $\xi^*_{\underline{\theta}}$. Hence, any equilibrium must satisfy the conditions of the Proposition 1.

Q.E.D.

Proof of Proposition 2.

The only possibility to disturb the contract is by offering a pooling one. Moreover it must also attract type $\overline{\theta}$ agents, so that there exists an amount of investment $\widetilde{I}$ satisfying

$$V^P_{\overline{\theta}}(\widetilde{F}, \widetilde{I}, p(\overline{\theta}, \widetilde{I}), b^j_t) \geq V^S_{\overline{\theta}}(F^{\prime j}_{\overline{\theta}}, I^{\prime j}_{\overline{\theta}}(b^j_t), p(\overline{\theta}, I^{\prime j}_{\overline{\theta}}), b^j_t) \text{ for } b_t \in [\underline{b}, I^*_{\underline{\theta}}).$$

In other words, there is no pooling contract that attracts all borrowers and earns a nonnegative expected profit if and only if (11) is satisfied.

The proof proceeds in five steps.

Step 1. *Pooling contract for a $\overline{\theta}$ type.*

Such a contract must obviously earn non-negative profits, i.e. $\widetilde{F} \geq \frac{R}{\widetilde{p(\theta, I)}}$ with $\widetilde{p(\theta, I)} = [\gamma B(\underline{\theta}) + (1-\gamma)B(\overline{\theta})](1 - e^{-\widetilde{I}})$. The most preferred pooling contract, for a $\overline{\theta}$ type, that is consistent with nonnegative expected profits for the bank has $\widetilde{F} = \frac{R}{\widetilde{p(\theta, I)}}$ and selects $\widetilde{I}$ such that

$$\widetilde{I} \equiv \arg\max_{\widetilde{I}} \{p(\overline{\theta}, \widetilde{I})[h^e G_{\overline{\theta}} - \widetilde{F}(\widetilde{I} - b^j_t)] + (1 - p(\overline{\theta}, \widetilde{I})h^u\}.$$

By FOC the amount of investment is,

$$\widetilde{I} = \ln\left(\frac{[\gamma B(\underline{\theta}) + (1-\gamma)B(\overline{\theta})](h^e G_{\overline{\theta}} - h^u)}{R}\right).$$

Step 2. *The indirect utility function of a high type under the pooling contract, $V^P_{\overline{\theta}}$, is decreasing in the proportion of low ability γ.*

If the pooling contract $(\widetilde{F}, \widetilde{I})$ is accepted by a talented borrower, her utility function is

$$V^P_{\overline{\theta}} = p(\overline{\theta}, \widetilde{I})[(h^e G_\theta - h^u) - \widetilde{F}(\widetilde{I} - b^j_t)] + h^u,$$

By substituting the contract $(\widetilde{F}, \widetilde{I})$ in the expression above, and taking the derivative we obtain,

$$\frac{dV^P_{\overline{\theta}}}{d\gamma} = B(\overline{\theta})[e^{-\widetilde{I}}(h^e G_\theta - h^u) - \widetilde{F}]\frac{d\widetilde{I}}{d\gamma} - B(\overline{\theta})\frac{d\widetilde{F}}{d\gamma}(\widetilde{I} - b^j_t).$$

The sign of the derivative is given by two terms. The first term cancels out by the envelop theorem, and the second term is positive since $\frac{d\widetilde{F}}{d\gamma} > 0$. As a consequence, $\frac{dV^P_{\overline{\theta}}}{d\gamma} < 0$.

Note that under a pooling contract the higher the number of low ability individuals, the higher the losses for the bank. Therefore, the higher is γ, the lower the probability of distorting the separating equilibrium. Since $V^P_{\overline{\theta}}$ is decreasing in the proportion of low ability (γ), we can argue that there will exist a $\widetilde{\gamma}$ such that when $\gamma \geq \widetilde{\gamma}$, type $\overline{\theta}$ is better under a separating contract (i.e., $V^P_{\overline{\theta}} < V^S_{\overline{\theta}}$), and thus the equilibrium is the separating one. Conversely, when $\gamma < \widetilde{\gamma}$, i.e. the probability of being a good type is high enough, $V^P_{\overline{\theta}} > V^S_{\overline{\theta}}$ holds and there is no equilibrium.

Step 3. *In a separating equilibrium the investment in college education for a talented borrower depends negatively on the level of inherited wealth, i.e.* $\frac{dI'^j_{\overline{\theta}}}{db^j_t} < 0$.

From the equation (7), and using the implicit function theorem, it holds that

$$\frac{dI'^j_{\overline{\theta}}}{db^j_t} = -\frac{R(\frac{B(\overline{\theta})-B(\underline{\theta})}{B(\overline{\theta})})}{-[B(\underline{\theta})e^{-I}(h^e G_{\underline{\theta}} - h^u) - \frac{B(\underline{\theta})}{B(\overline{\theta})}R]} = \frac{\beta R(\frac{B(\overline{\theta})-B(\underline{\theta})}{B(\overline{\theta})})}{(1-\gamma)[B(\overline{\theta})e^{-I}(h^e G_{\overline{\theta}} - h^u) - R]},$$

where β is the multiplier associated with $IC_{\underline{\theta}}$(equation (7)), which is binding, and therefore $\beta > 0$. With overinvestment (that is $\acute{I}^j_{\overline{\theta}} > I^*_{\overline{\theta}}$) we obtain $B(\overline{\theta})e^{-I}(h^e G_{\overline{\theta}} - h^u) < R$. Therefore, $\frac{dI'^j_{\overline{\theta}}}{db^j_t} < 0$.

In fact,

$$\beta = (1-\gamma)\frac{B(\overline{\theta})e^{-I}(h^e G_{\overline{\theta}} - h^u) - R}{B(\underline{\theta})e^{-I}(h^e G_{\underline{\theta}} - h^u) - R\frac{B(\underline{\theta})}{B(\overline{\theta})}} = (1-\gamma)\frac{B(\overline{\theta})e^{-I}(h^e G_{\overline{\theta}} - h^u) - R}{\frac{B(\underline{\theta})}{B(\overline{\theta})}[B(\overline{\theta})e^{-I}(h^e G_{\underline{\theta}} - h^u) - R]}. \quad (22)$$

Step 4. *The population share* $\widetilde{\gamma}$ *changes with the inherited wealth. In particular, it is decreasing with* b^j_t.

By definition $\widetilde{\gamma}$ is implicitly defined by,

$$V^P_{\overline{\theta}}(\widetilde{F}(\widetilde{\gamma}), \widetilde{I}(\widetilde{\gamma}), p(\overline{\theta}, \widetilde{I}), b^j_t) = V^S_{\overline{\theta}}(F'^j_{\overline{\theta}}, I'^j_{\overline{\theta}}(b^j_t), p(\overline{\theta}, I'^j_{\overline{\theta}}), b^j_t) \text{ for any } b_t \in [\underline{b}, I^*_{\underline{\theta}}).$$

Let us denote this equation by $\ell = V^s_{\overline{\theta}} - V^P_{\overline{\theta}}$. By the implicit function theorem $\frac{d\widetilde{\gamma}}{db_t} = -\frac{\frac{d\ell}{db}}{\frac{d\ell}{d\widetilde{\gamma}}}$. The denominator is positive since $\frac{dV^P_{\overline{\theta}}}{d\gamma}$<0, and the sign of this expression is given by the sign of the numerator

$$\frac{d\ell}{db} = R(1 - \frac{B(\overline{\theta})}{(1-\gamma)B(\overline{\theta}) + \gamma B(\underline{\theta})}) + [B(\overline{\theta})(h^e G_{\overline{\theta}} - h^u) - R]\frac{dI'^j_{\overline{\theta}}}{db^j_t},$$

From Step 3 we can rewrite this expression as

$$\frac{d\ell}{db} = \frac{R(B(\overline{\theta}) - B(\underline{\theta}))}{[(1-\gamma)B(\overline{\theta}) + \gamma B(\underline{\theta})]B(\overline{\theta})(1-\gamma)}[-\gamma(1-\gamma)B(\overline{\theta}) + \beta((1-\gamma)B(\overline{\theta}) + \gamma B(\underline{\theta}))],$$

where β is given by (22).

The expression below is positive:

$x = -\gamma(1-\gamma)B(\overline{\theta}) + \beta\{(1-\gamma)B(\overline{\theta}) + \gamma B(\underline{\theta})\} = \beta(1-\gamma)B(\overline{\theta}) + (1-\gamma)\gamma B(\overline{\theta})\{\frac{B(\overline{\theta})e^{-I}(h^e G_{\overline{\theta}} - h^u) - R}{B(\overline{\theta})e^{-I}(h^e G_{\underline{\theta}} - h^u) - R} - 1\} > 0$

Consequently $\frac{d\widetilde{\gamma}}{db_t^j} < 0$. As a result, we need the maximum level of $\widetilde{\gamma}(\underline{b})$ to be $0 < \widetilde{\gamma}(\underline{b}) < 1$. Therefore, if $\gamma > \widetilde{\gamma}(\underline{b})$ the unique equilibrium is the separating one since the break-even line ($\widetilde{F} = \frac{R}{\widetilde{p(\theta, I)}}$) does not touch the pooling area (this region is given by the intersection of the indifference curves of $\overline{\theta}$ and $\underline{\theta}$ that pass through the points $\xi^*_{\underline{\theta}}$ and $\xi^{'j}_{\overline{\theta}}$).

Step 5. *Note that for* $\widetilde{\gamma}(\underline{b}) = 0, V^s_{\overline{\theta}} - V^p_{\overline{\theta}} < 0$. *On the other hand, for* $\widetilde{\gamma}(\underline{b}) = 1$ *we obtain that* $V^s_{\overline{\theta}} - V^p_{\overline{\theta}} > 0$. *Then, since the function* $V^s_{\overline{\theta}} - V^p_{\overline{\theta}}$ *is continuous in* $\widetilde{\gamma}(\underline{b})$, *we apply the Intermediate Value Theorem (I.V.T) to argue that there exists a level of* $\widetilde{\gamma}(\underline{b})$ *such that* $0 < \widetilde{\gamma}(\underline{b}) < 1$.

The value of $\widetilde{\gamma}(\underline{b})$ is given by

$$p(\overline{\theta}, I^{'}_{\overline{\theta}})(h^e G_{\overline{\theta}} - h^u) - RI^{'}_{\overline{\theta}} + R\underline{b} = p(\overline{\theta}, \widetilde{I}(\widetilde{\gamma}(\underline{b})))[(h^e G_{\overline{\theta}} - h^u) - \widetilde{F}(\widetilde{\gamma}(\underline{b}))(\widetilde{I}(\widetilde{\gamma}(\underline{b})) - \underline{b})]. \tag{23}$$

Let us study the possible extreme values that $\widetilde{\gamma}(\underline{b})$ may take. First, we analyze the limit case when $\widetilde{\gamma}(\underline{b})$ takes value 0. Then, $lim_{\widetilde{\gamma}\to 0}\widetilde{I}_{\overline{\theta}} = I^*_{\overline{\theta}}$ and the indirect utility function under the pooling contract becomes the indirect utility under full information ($lim_{\widetilde{\gamma}\to 0}V^p_{\overline{\theta}} \equiv V^*_{\overline{\theta}}$). Therefore, $V^s_{\overline{\theta}} - V^p_{\overline{\theta}} < 0$ because the indirect utility under full information is always higher than the one under asymmetric information.

Second, we analyze the case when in the limit $\widetilde{\gamma}$ takes value 1. Then, $lim_{\widetilde{\gamma}\to 1}\widetilde{I} = \ln(\frac{B(\underline{\theta})(h^e G_{\overline{\theta}} - h^u)}{R})$ and under $A3$ we obtain $V^s_{\overline{\theta}} - V^p_{\overline{\theta}} > 0$.

Since $A3$ holds, we have $V^s_{\overline{\theta}} - V^p_{\overline{\theta}} > 0$. That is,

$p(\overline{\theta}, I^{'j}_{\overline{\theta}})(h^e G_{\overline{\theta}} - h^u) - RI^{'j}_{\overline{\theta}} + R\underline{b} > p(\overline{\theta}, \widetilde{I})[(h^e G_{\overline{\theta}} - h^u) - \widetilde{F}(\widetilde{I} - \underline{b})]$.

From the $IC_{\underline{\theta}}$ of the low type (equation (7)) we have

$-R(I^{'j}_{\overline{\theta}} - \underline{b}) = B(\overline{\theta})(h^e G_{\underline{\theta}} - h^u)\{-e^{-I^*_{\underline{\theta}}} + e^{-I^{'}_{\overline{\theta}}}\} - R\frac{B(\overline{\theta})}{B(\underline{\theta})}[\ln(\frac{d}{R}) - \underline{b}]$.

After some substitution:

$B(\underline{\theta})e^{-I^{'j}_{\overline{\theta}}}\{(h^e G_{\underline{\theta}} - h^u) - (h^e G_{\overline{\theta}} - h^u)\} < R\ln(\frac{h^e G_{\overline{\theta}} - h^u}{h^e G_{\underline{\theta}} - h^u})$,

The only endogenous variable in this inequality is $I^{'j}_{\overline{\theta}}$, and note that the LHS is decreasing in $I^{'j}_{\overline{\theta}}$. Then, if we substitute the value of $I^{'j}_{\overline{\theta}}$ with the first best contract for the high type, this inequality holds for sure for a level of investment higher than the first best. In other words, once we substitute with $I^*_{\overline{\theta}}$, the inequality above becomes true when $A3$ holds.

Q.E.D.

Proof of Proposition 3.

To determine the existence of a unique invariant distribution G we need to study the long run dynamic behavior implied by the transition function $P(\cdot)$. This transition function is defined as follows,

Definition 1 Let Σ denote the set of Borel subsets of $\beta = [\underline{b}, \overline{b}]$. A transition function on a measurable interval A is a mapping such that $P : \Sigma \times \beta \to [0, 1]$. That is

$$P(b, A) = P(b_{t+1} \in A/b_t = b), \quad \text{for all Borel subsets } A \in \Sigma,$$

where $b_{t+1} = g(b_t, \overline{\theta}, \delta)$ and $P(b, A)$ is the probability that the next period's bequest lies in the set A given that the current bequest is b.

In our model, the law of motion of the bequest defines a Markov chain with a transition function P given by

$$\begin{aligned} p(b, A) \quad = \quad & \gamma[(1 - \underline{p})I_A(g(b, \underline{\theta}, 0)) + \underline{p}I_A(g(b, \underline{\theta}, 1))] + \\ & (1 - \gamma)[(1 - \overline{p})I_A(g(b, \overline{\theta}, 0)) + \overline{p}I_A(g(b, \overline{\theta}, 1))], \end{aligned} \tag{24}$$

where

$$I_A(i) = \begin{cases} 1 & if \qquad\qquad i \in A \\ 0 & otherwise. \end{cases}$$

Following theorem 8.1 of Stokey and Lucas (1989), associated with any transition function on a measurable space (A, Σ) there is an operator on a probability measure. For any probability measure μ on (A, Σ) define $T^*\mu$ by

$$(T^*\mu)(A) = \int P(b, A)\mu(db), \quad \text{all } A \in \Sigma. \tag{25}$$

The operator T maps the probability measure onto itself, so that $T^*\mu$ is the probability measure over the state of the next period if μ is the probability measure over the current state. The sequence of distribution functions of the bequest $\{G\}_{t=1}^{\infty}$ is given inductively by equation (25), where the distribution G_0 is simply a mass point at the beginning of the time.

We would like to know if the mapping T is a contraction mapping, having a fixed point. Before that, however, we define a stationary distribution of wealth.

Definition 2 A wealth distribution $G(b)$ on β is invariant for P if for all Borel subsets $A \subset \Sigma$, one has the equality

$$T^*G(A) = G(A).$$

We apply Hopenhayn and Prescott's (1992) analysis of existence, uniqueness and convergence properties of monotonic stochastic processes.

A. *Existence*.

Proof. The existence of an invariant distribution G^{FI} for the Markov process follows immediately from the monotonicity of P established in Hopenhayn-Prescott's Corollary 4.

From this corollary, the only condition that has to hold is the monotonicity of the transition probability $P(b,.)$. In our model the transition function $p(b,A)$ is increasing in its first argument b in the following first-order stochastic dominance sense: for all $(b,b')\in \beta^2$, $b \leq b'$ implies for any $x \in B$,

$$p(b',[\underline{b},x]) \leq p(b,[\underline{b},x]).$$

More specifically we obtain

$$p(b',[\underline{b},x]) - p(b,[\underline{b},x]) = \gamma[(1-\underline{p})\{I_A(g(b',\underline{\theta},0)) - I_A(g(b,\underline{\theta},0))\}$$

$$+\underline{p}\{I_A(g(b',\underline{\theta},1)) - I_A(g(b,\underline{\theta},1))\}] + (1-\gamma)[(1-\overline{p})\{I_A(g(b',\overline{\theta},0))$$

$$-I_A(g(b,\overline{\theta},0)\} + \overline{p}\{I_A(g(b',\overline{\theta},1)) - I_A(g(b,\overline{\theta},1))\}] \leq 0,$$

where $A = [0,x]$. Notice that it is negative since $g(b',.,.) \geq g(b,.,.)$, and therefore $I(g(b',.,.)) - I(g(b,.,.))$ takes either the value of -1 or 0.

B. *Uniqueness and Convergence.* ■

Proof. The proof of the uniquess and convergence of G^{FI} follows directly from the Hopenhayn-Prescott's Theorem 2.

If the linear Markov process satisfies the monotonicity property as well as the following "concavity property" or "Monotone Mixing Conditions" (in short MMC), then there is a unique and convergent distribution G^{FI}.

The "MMC" or "concavity property" tells us that we can find a point $b \in [\underline{b},\overline{b}]$ such that there exists an $n \geq 1$ and $\epsilon > 0$, such that in n good realizations of the shock we have $p(\underline{b},[b^*,\overline{b}])^n > \epsilon$, and in n bad realizations of the shock we have $p(\overline{b},[b^*,\underline{b}])^n > \epsilon$.

In our model, we can find $n = 1$ and $0 < \epsilon < 1$ such that

$$\begin{aligned} p(\underline{b},[b^*,\overline{b}]) &= (1-\gamma)\underline{p} \text{ for the low type and} \\ p(\underline{b},[b^*,\overline{b}]) &= \gamma\overline{p} \text{ for the high type.} \end{aligned}$$

Similarly,

$$\begin{aligned} p(\overline{b},[b^*,\underline{b}]) &= (1-\gamma)(1-\underline{p}) \text{ for the low type and} \\ p(\overline{b},[b^*,\underline{b}]) &= \gamma(1-\overline{p}) \text{ for the high type.} \end{aligned}$$

By definition $\underline{p}$ and $\overline{p}$ are probabilities and thus $\lim_{I \to 0} \overline{p} = B(\overline{\theta}) < 1$ and $\lim_{I \to \infty} \underline{p} = 0$.

Q.E.D. ■

Proof of Proposition 4.

As the success probability is decreasing in the inherited wealth, the transition probability P under asymmetric information is not monotonic in the first order stochastic dominance sense. As a result, we cannot apply Hopenhayn-Prescott's Corollary 4. In this case, in order to prove the existence, uniqueness and convergence of an invariant distribution we show that our transition probability satisfies *Condition M* in Section 11.4 of Stokey and Lucas (1989). That is

Condition M: There exists $\varepsilon > 0$ and an integrer $n \geq 1$ such that for any $A \in \chi$, either $p^n(s, A) \geq \varepsilon$, or $p(s, A^c)^n \leq \varepsilon$,for all $s \in S$.

For any $b_t < I^*_{\underline{\theta}}$ the transition function with asymmetric information is

$$\begin{aligned} p(b, A) &= \gamma[(1-\underline{p})I_A(g(b, \underline{\theta}, 0)) + \underline{p}I_A(g(b, \underline{\theta}, 1))] \\ &\quad +(1-\gamma)[(1-\overline{p}')I_A(g(b, \overline{\theta}, 0)) + \overline{p}' I_A(g(b, \overline{\theta}, 1))], \end{aligned}$$

where

$$I_A(i) = \begin{cases} 1 & if \qquad\qquad i \in A \\ 0 & otherwise, \end{cases}$$

and $\overline{p}' = p(\overline{\theta}, I^{'}_{\overline{\theta}}(b))$. If the wealth is $b \geq I^*_{\underline{\theta}}$, the transition function coincides with the full information one (see equation (26)).

The complementary of A is denoted by A^c. If $\underline{b} \in A$,

$$P(b, A) = (1-\gamma)(1 - p(\overline{\theta}, I^{'}_{\overline{\theta}}(b_t))) + \gamma(1 - p(\underline{\theta}, I^*_{\underline{\theta}})) = 1 - \varepsilon_1 \text{ if } b \leq I^*_{\underline{\theta}},$$

and

$$P(b, A) = (1-\gamma)(1 - p(\overline{\theta}, I^*_{\overline{\theta}})) + \gamma(1 - p(\underline{\theta}, I^*_{\underline{\theta}})) = 1 - \varepsilon_2 \text{ if } b > I^*_{\underline{\theta}}.$$

Similarly, if $\underline{b} \in A^c$

$$P(b, A^c) = (1-\gamma)p(\overline{\theta}, I^{'}_{\overline{\theta}}(b_t)) + \gamma p(\underline{\theta}, I^*_{\underline{\theta}}) = \varepsilon_1 \text{ if } b \leq I^*_{\underline{\theta}},$$

and

$$P(b, A^c) = (1-\gamma)p(\overline{\theta}, I^*_{\overline{\theta}}) + \gamma p(\underline{\theta}, I^*_{\underline{\theta}}) = \varepsilon_2 \text{ if } b > I^*_{\underline{\theta}}.$$

Letting $\varepsilon = \max(\varepsilon_1, \varepsilon_2, 1-\varepsilon_1, 1-\varepsilon_2)$, we have that *Condition M* holds, and Theorem 11.12 (which tells us about the convergence of the probability measures) is also satisfied.

Q.E.D.

Proof Proposition 5.

Since $p(E)^{AI}$ (or similarly $1 - G^{AI}(x)$) is given by the equation (21) and $p(E)^{FI}$ (i.e., $1 - G^{FI}(x)$) is given in the equation (15), it is easy to show that

$$p(E)^{AI} - p(E)^{FI} = (1-\gamma) \int_{\underline{b}}^{I^*_{\underline{\theta}}} [p(\overline{\theta}, I'_{\underline{\theta}}(b)) - \overline{p}] dG^{AI}(b) > 0.$$

Since $p(U)^{FI} + p(E)^{FI} = p(U)^{AI} + p(E)^{AI} = 1$, then $p(U)^{AI} < p(U)^{FI}$.
Q.E.D.

Proof of Proposition 6.

The number of uneducated agents can also be defined as $1 - G(x)$, where $x = \alpha[h^u + R(\overline{b} - I^*_{\underline{\theta}})]$ is the highest second period wealth of an uneducated lender. Clearly, x could be bigger or smaller than $I^*_{\underline{\theta}}$.

Let us first consider $x < I^*_{\underline{\theta}}$. In this case all the children of uneducated people will be borrowers. Moreover, all talented borrowers (with inherited wealth belonging to the interval $[\underline{b}, I^*_{\underline{\theta}})$) will receive overinvestment.

We now compute upward mobility under full and asymmetric information.

$p(E/U)^{AI} = \frac{1}{G^{AI}(x)}[(1-\gamma)\int_{\underline{b}}^{x} p(\overline{\theta}, I'_{\underline{\theta}}(b)) dG^{AI}(b) + \gamma \underline{p} G^{AI}(x)]$.

$p(E/U)^{FI} = (1-\gamma)\overline{p} + \gamma \underline{p}$.

The difference is

$p(E/U)^{AI} - p(E/U)^{FI} = \frac{(1-\gamma)}{G^{AI}(x)}[\int_{\underline{b}}^{x}\{p(\overline{\theta}, I'_{\underline{\theta}}(b)) - \overline{p}\} dG^{AI}(b)] > 0$

Analogously we can show that the result is unaffected if $x \geq I^*_{\underline{\theta}}$.
Q.E.D.

Appendix B.

B.1. *Indifferences curves are concave in the plane (I, F).*

The expected utility of the borrower is given by

$$U_{\theta,b_t} = p(\theta, I)[h^e G_\theta - F(I - b^j_t)] + (1 - p(\theta, I))h^u.$$

and the slope of her indifference curve is

$$\frac{dF}{dI} = -\frac{\frac{dU}{dI}}{\frac{dU}{dF}} = -\frac{[\frac{dp(\theta,I)}{dI}(h^e G_\theta - h^u) - F(I - b_t)] - p(\theta, I)F}{-p(\theta, I)(I - b^j_t)}. \tag{26}$$

When $I = b^j_t$ the slope is not defined. The demand curve $I = I(F)$, which is decreasing in the plane (F, I), is given by

$$\frac{dp(\theta, I)}{dI}[(h^e G_\theta - h^u) - F(I - b_t)] - p(\theta, I)F = 0.$$

The slope of the indifference curve will be zero if and only if (F, I) satisfies the demand function. To obtain information on the shape of the indifference curve for points not on the demand curve, we differentiate the equation (26) with respect to I and arrange terms as follows,

$$\frac{d^2F}{dI^2} = \frac{[-B(\theta)e^{-I}((h^eG_\theta - h^u) - F(I-b_t^j)) - 2B(\theta)e^{-I}F]}{p(\theta,I)^2(I-b_t^j)^2} - \frac{[\frac{dp(\theta,I)}{dI}((h^eG_\theta - h^u) - F(I-b_t^j)) - p(\theta,I)F][B(\theta)e^{-I}(I-b_t^j) + P(\theta,I)]}{p(\theta,I)^2(I-b_t^j)^2}.$$

The denominator is positive, so the sign is determined by the two terms in the numerator. Since $p_{II} < 0$ the first term is negative. The second term is of uncertain sign, but includes the slope of the indifference curve as a multiplicative element. Consequently, we know that where the indifference curve has zero slope, that is, where it intersects the demand function, it must have a negative second derivative. Thus, in the neighborhood of the demand function, the second term cancels out and the indifference curve is concave. We know, however, that the slope of the indifference curve can change sign only at the point of intersection with the demand curve. The result, therefore, is that the indifference curves are monotonically increasing until they reach the demand function and monotonically decreasing thereafter (see Figure 1).

It is worth noticing that the single crossing property (that is $\frac{dF}{dI}\mid_{\overline{\theta}} > \frac{dF}{dI}\mid_{\underline{\theta}}$) holds for every $b_t^j < I_\theta^*$. Specifically, in this region the marginal decrease in the interest rate F that a borrower is willing to accept in order to receive a higher I (to be near to the efficient amount) is higher for type $\underline{\theta}$. This implies that an increase in I does less harm to the high type, which is the one that is distorted in equilibrium.

Finally, we can easily check that an increase in the inherited wealth increases the slope of an indifference curve at a specific interest rate-investment point, i.e. $\frac{d}{db_t}(\frac{dF}{dI}) > 0$. Consider a borrower with ability θ; it is easy to check that $\frac{dF}{dI}\mid_{b_t^j} > \frac{dF}{dI}\mid_{b_t^i}$holds for every $b_t^j < b_t^j < I_\theta^*$. That is, the slope is steeper for a higher amount of inherited wealth. This means that the higher the bequest is, the more inclined an individual is to accept an increase in the investment for a certain reduction in the interest rate. Note that in the model money is a substitute of intelligence.

Q.E.D

B.2. *The isoprofit line is a decreasing and convex curve in the plane* (I, F). The profit of the bank is

$$\Pi = p(\theta,I)F(I-b_t^j) - R(I-b_t^j).$$

Defining the function $F(I)$ we have

$$F = \frac{R(I-b_t^j) + \Pi}{p(\theta,I)(I-b_t^j)}.$$

Because of perfect competition in equilibrium banks make zero profits ($\Pi =$

0), and thus the slope becomes decreasing

$$\frac{dF}{dI} = -\frac{B(\theta)e^{-I}R}{p(\theta, I)^2} < 0.$$

The isoprofit is a convex function,

$$\frac{d^2F}{d^2I} = \frac{B(\theta)e^{-I}Rp(\theta, I)(p(\theta, I) + 2B(\theta)e^{-I})}{p(\theta, I)^4} > 0.$$

The break-even line for talented agents are on the left of $\pi_{\underline{\theta}}$, since those borrowers have a higher probability of success.

Finally, the break-even line $F = \frac{R}{p(\theta,I)}$ satisfies $\lim_{I\to 0} F = \infty$, and $\lim_{I\to\infty} F = \frac{R}{B(\theta)}$.

Q.E.D.

B.3. *With full information contracts* $IC_{\overline{\theta}} > 0$ *for any* $b_t^j \in [\underline{b}, I_{\underline{\theta}}^*)$.

Notice first that $\frac{dIC_{\overline{\theta}}}{db_t} < 0$, and $\frac{dIC_{\underline{\theta}}}{db_t} > 0$. Therefore, we just need to be sure that in the limit $b_t = I_{\underline{\theta}}^*$, the incentive compatible for the high type is satisfied while the one for the low type is not.

For talented borrowers who inherit the first best amount of investment, in the limit $b_t = I_{\underline{\theta}}^*$ the incentive compatibility becomes

$$IC_{\overline{\theta}}\left(I_{\underline{\theta}}^*\right) = 1 + \ln(\frac{a}{d}) < \frac{a}{d} \tag{27}$$

Since $a > d$, $1 + \ln(\frac{a}{d}) < \frac{a}{d}$ is always true.

Q.E.D.

B.4. *At any level of inherited wealth the investment is characterized by maximum equity participation.*

Under full information agents receive the optimal contract, and thus it is always optimal to put all their wealth in the investment in education.

We know from proposition 1 that contract for $\overline{\theta}$ agents are distorted, and thus, they are worse off than under full information. The utility of the talented agents are

$$U_{\overline{\theta},b_t^j} = p(\overline{\theta}, I_{\overline{\theta}}^{'})[(h^e G_{\overline{\theta}} - h^u) - F_{\overline{\theta}}^{'}(I_{\overline{\theta}}^{'} - I_{\underline{\theta}}^*)] + (1 - p(\overline{\theta}, I_{\overline{\theta}}^{'}))h^u.$$

We can see that the best thing to do is to put the inherited wealth in the investment since

$$\frac{dU_{\overline{\theta}}}{db_t^j} = [B(\overline{\theta})e^{-I_{\overline{\theta}}^{'}}(h^e G_{\overline{\theta}} - h^u) - R]\frac{dI_{\overline{\theta}}^{'}}{db_t^j} + R > 0,$$

notice that since $I_{\overline{\theta}}^{'} > I_{\overline{\theta}}^*$ then $B(\overline{\theta})e^{-I}(h^e G_{\overline{\theta}} - h^u) < R$ and $\frac{dI}{db_t^j} < 0$.

Q.E.D.

B.5. *With asymmetric information, first, high ability borrowers prefer the contract offered to their type instead of the contract offered to the low ability agents, namely* $IC_{\overline{\theta}} > 0$. *Second, high ability agents with* $b_t^j \in [\underline{b}, I^*_{\underline{\theta}})$ *prefer to become borrowers rather than self-financed.*

Stept 1. It is eassy to see that under asymmetric information the derivative of the $IC_{\overline{\theta}}$ is decreasing in the level of bequest. That is, $\frac{\partial U_{\overline{\theta},\overline{\theta}}}{\partial b_t} < \frac{\partial U_{\overline{\theta},\underline{\theta}}}{\partial b_t}$, where $U_{\overline{\theta},\underline{\theta}}(b_t)$ represents the utility of a talented agent when she chooses the low type contract, while $U_{\overline{\theta,\theta}}(b_t)$ is her utility when she choose the high type contract.

Sept 2. We can show that $\frac{\partial U_{\overline{\theta},\underline{\theta}}}{\partial b_t} < \frac{\partial U^A_{\overline{\theta}}}{\partial b_t}$, where $U^A_{\overline{\theta}}(b_t)$ represents the utility of a talented agent when she does not go to the capital market. The utility of a talented agent when she chooses the high type contract is

$U_{\overline{\theta},\underline{\theta}} = B(\overline{\theta})(h^e G_{\overline{\theta}} - h^u)(1 - e^{-I^*_{\underline{\theta}}}) - R\frac{B(\overline{\theta})}{B(\underline{\theta})}(I^*_{\underline{\theta}} - b_t^j) + h^u$

Similarly, $U^A_{\overline{\theta}}(b_t)$ is given by

$U^A = B(\overline{\theta})(h^e G_{\overline{\theta}} - h^u)(1 - e^{-b_t^j}) + h^u$

Thus, it is easy to chek that

$\frac{\partial\left(U_{\overline{\theta},\underline{\theta}} - U^A_{\overline{\theta}}\right)}{\partial b_t} = \frac{B(\overline{\theta})}{B(\underline{\theta})}[-B(\overline{\theta})(h^e G_{\overline{\theta}} - h^u)e^{-b_t^j} + R] < 0.$

When $b_t^j = \ln(\frac{B(\underline{\theta})(h^e G_{\overline{\theta}} - h^u)}{R})$ we have that $\frac{\partial\left(U_{\overline{\theta},\overline{\theta}} - U^A_{\overline{\theta}}\right)}{\partial b_t} = 0.$

Notice that a borrower is an agent with $b_t^j < I^*_{\underline{\theta}} < \ln(\frac{B(\underline{\theta})(h^e G_{\overline{\theta}} - h^u)}{R})$. Therefore, $-B(\overline{\theta})(h^e G_{\overline{\theta}} - h^u)e^{-b_t^j} > R$ and thus $\frac{\partial\left(U_{\overline{\theta},\underline{\theta}} - U^A_{\overline{\theta}}\right)}{\partial b_t} < 0.$

Stept 3. From stept 1 and 2 we obtain $\frac{\partial U_{\overline{\theta},\overline{\theta}}}{\partial b_t} < \frac{\partial U_{\overline{\theta},\underline{\theta}}}{\partial b_t} < \frac{\partial U^A_{\overline{\theta}}}{\partial b_t}$. Therefore, $\frac{\partial U_{\overline{\theta},\overline{\theta}}}{\partial b_t} < \frac{\partial U^A_{\overline{\theta}}}{\partial b_t}$.

Therefore since the function $U_{\overline{\theta},\overline{\theta}} - U^A_{\overline{\theta}}$ is monotonically decreasing in b_t^j, then it is enough to show that $U_{\overline{\theta},\overline{\theta}}(b_t^j = I^*_{\underline{\theta}}) > U^A_{\overline{\theta}}(b_t^j = I^*_{\underline{\theta}})$ to proof that $IC_{\overline{\theta}} > 0$ for any $b_t^j \in [\underline{b}, I^*_{\underline{\theta}})$.

The value of $U_{\overline{\theta},\overline{\theta}}(b_t^j = I^*_{\underline{\theta}})$ is

$U_{\overline{\theta},\overline{\theta}}(b_t^j = I^*_{\underline{\theta}}) = p(\overline{\theta}, I'_{\overline{\theta}})(h^e G_{\overline{\theta}} - h^u) - R(I'_{\overline{\theta}} - I^*_{\underline{\theta}}).$

Similarly the value of $U^A_{\overline{\theta}}(b_t^j = I^*_{\underline{\theta}})$ becomes

$U^A_{\overline{\theta}}(b_t^j = I^*_{\underline{\theta}}) = p(\overline{\theta}, I^*_{\underline{\theta}})(h^e G_{\overline{\theta}} - h^u).$

Note that the expresion of $U^A_{\overline{\theta}}(b_t^j = I^*_{\underline{\theta}})$ coincide with the value of $U_{\overline{\theta},\overline{\theta}}(b_t^j = I^*_{\underline{\theta}})$. Therefore, to proof that high ability borrowers prefer the contract for their type and to proof that high ability borrowers do not become self finance is the same. So we just need to be sure that the following inequality holds,

$$p(\overline{\theta}, I'_{\overline{\theta}})(h^e G_{\overline{\theta}} - h^u) - R(I'_{\overline{\theta}} - I^*_{\underline{\theta}}) > p(\overline{\theta}, I^*_{\underline{\theta}})(h^e G_{\overline{\theta}} - h^u), \tag{28}$$

From equation (7) we know that $-R(I'_{\overline{\theta}} - I^*_{\underline{\theta}}) = B(\overline{\theta})(h^e G_{\underline{\theta}} - h^u)\{(1 - e^{-I^*_{\underline{\theta}}}) -$

$(1 - e^{-I'_{\overline{\theta}}})\}$. If we substituting this value in equation (28) we obtain

$$B(\overline{\theta})(h^e G_{\overline{\theta}} - h^u) > B(\overline{\theta})(h^e G_{\underline{\theta}} - h^u),$$

which always holds.

Q.E.D.

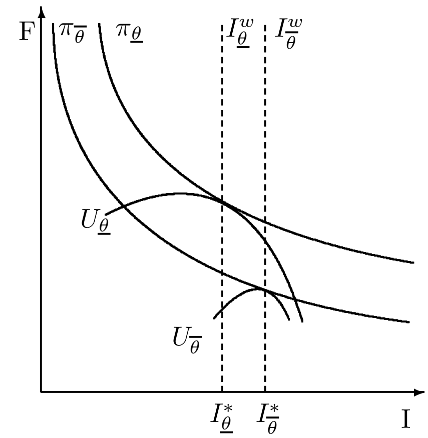

Figure 1: Equilibrium with full information

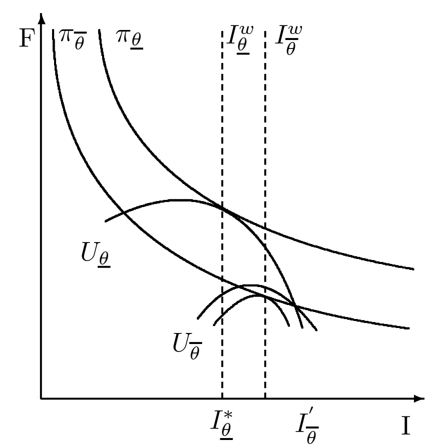

Figure 2: Equilibrium with asymmetric information

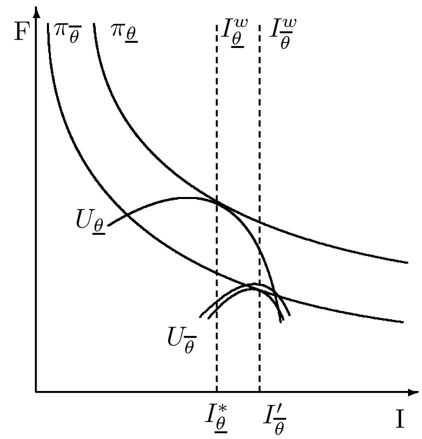

Figure 3: Equilibrium with asymmetric information

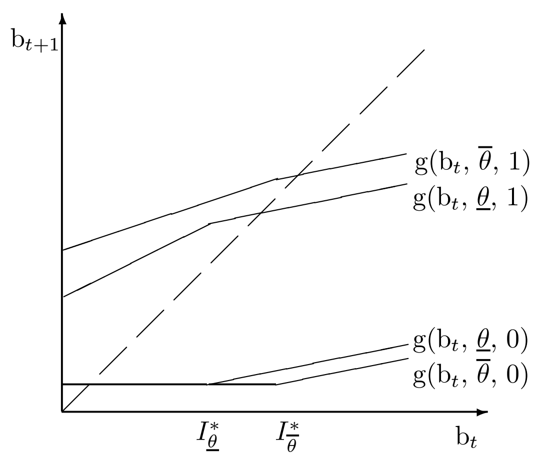

Figure 4: Individual transition function with full information